# Standard Errors in the Consumer Expenditure Survey

### JEFFREY L. BLAHA

ata for the Consumer Expenditure (CE) Survey are collected from a sample of all the consumer units (CUs) in the United States. Estimates of the average (mean) annual expenditure per CU in the CE tables for the year 2000 were based on a sample of about 30,000 CUs, out of a total of about 109 million in the Nation. These mean estimates differ from the true population values because a subset, rather than the whole population, is observed. Sampling error is the difference between the survey estimate and the true population value. The most common measure of the magnitude of the sampling error is the standard error of the estimate. The standard error provides data users with information about the variability associated with the estimate.

Prior to the publication of the 2000 data, the CE program made available separate tables of standard errors for the Interview and Diary components of the CE Survey. Starting with the 2000 data, the CE program began using tables with integrated data from both surveys. Integrated data provide a complete accounting of consumer expenditures and income, which neither survey component alone is designed to do. The tables, which correspond to standard integrated tables of CU expenditures published in the CE reports and on the CE Web site, are provided by standard demographic characteristics

(except for region).<sup>1</sup> This article gives a summary description of the halfsample replication method used to calculate the standard error statistics and demonstrates the proper interpretation of these statistics.

#### Methodology

Standard textbook formulas for calculating standard errors assume simple random sampling and do not apply to the CE Survey, because it does not use a simple random sample. Instead, the Survey uses stratified random sampling, with systematic sampling within the strata. Hence, a different method for calculating standard errors is needed. Replication methods make up a class of techniques that provides a way to produce unbiased and designconsistent estimates of standard error for complex survey designs when the usual assumptions are not satisfied. The fundamental idea behind replication methods is to select subsamples repeatedly from the full sample, calculate the statistic of interest for each subsample, and use the variability among the subsamples to estimate the standard error of the full-sample statistic.

Jeffrey L. Blaha is a mathematical statistician in the Survey Methods Division, Consumer Expenditure Survey, Bureau of Labor Statistics.

<sup>&</sup>lt;sup>1</sup>The replication methodology used to calculate the standard errors is designed to work at the national level and is not applicable to regional estimates.

The balanced repeated replication method is used to estimate the standard error in the CE Survey. In this method, sampled geographic locations are divided into 40 groups (called strata). The CUs within each stratum are randomly divided into two half samples. Half of the CUs are assigned to one half sample, and the other half are assigned to the other half sample. Because there are 40 strata and 2 groups of CUs in each stratum, we can compute 240 (approximately 18 trillion) different estimates of expenditure in which we use exactly half of the collected data. With this information, we can estimate the standard error of CE estimates by examining how the different estimates compare with the full-sample estimate. In the balanced repeated replication method, we use a  $44 \times 44$ Hadamard matrix to choose the 44 "best" combinations of groups out of the 18 trillion possible combinations.

A *variance* estimate for each category of item is obtained by first computing the mean estimate of the item for each replicate, then summing the squared deviations of the replicate mean estimates from the full-sample mean estimate, and then dividing by the number of replicates. Thus,

$$V(\bar{x}_{i}) = \frac{1}{44} \sum_{r=1}^{44} (\bar{x}_{ir} - \bar{x}_{i})^{2},$$

where  $x_{ir}$  is a calendar-period estimate of the mean expenditure for item *i*, using the rth replicate data and  $x_i$  is the calendar-period estimate of the mean expenditure for item *i*, using the fullsample data.

The standard error is calculated as

the square root of the variance:

$$\operatorname{SE}(\overline{x}_i) = \sqrt{V(\overline{x}_i)}.$$

The *coefficient of variation* (CV) is the standard error expressed as a percentage of the sample mean estimate and thus is independent of the scale of measurement. The CV is calculated as

$$\operatorname{CV}(\overline{x}_i) = \frac{\operatorname{SE}(\overline{x}_i)}{\overline{x}_i} \times 100.$$

These formulas apply to aggregated categories as well as individual component items. In producing a table that uses integrated data from both surveys, the aggregated categories may be composed of items from one survey or the other, or they can be based on integrated data from both surveys.

#### Interpretation of the statistics

The primary purpose of calculating standard errors for the mean estimates is to provide data users with a measure of the variability associated with the estimates. This variability measures how close different estimates would be to each other if it were possible to repeat the survey over and over, using different samples of CUs. While it is not feasible to repeat the survey over and over, statistical theory allows the standard error to be estimated anyway. A small standard error indicates that multiple independent samples would produce values that are consistently very close to each other, whereas a large standard error indicates that multiple independent samples would produce values that are consistently not very close to each other.

Table 1 is an extract from one of the standard published CE demographic tables. The table shows the mean estimate, standard error, and coefficient of variation for a list of expenditure items and categories, using integrated data from both the Interview and Diary Surveys in 2000. For example, the table shows that the average annual expenditure by all CUs on personal care products and services for 2000 was \$563.62, with a standard error of \$7.94. Because it was impossible to ask all CUs in the country how much they spent on personal care products and services, the \$563.62 mean figure is an estimate, and we have a margin of error, usually defined as  $\pm 2$  standard errors. In this example, the average annual expenditure on personal care products and services has a margin of error of  $\pm$  \$15.88. Thus, we can say that the average CU probably spent between \$547.74 and \$579.50  $($563.62 \pm $15.88)$  annually on personal care products and services.

Because the CV is the standard error as a proportion of the mean estimate, it provides an indication of the spread of the data around the mean. The smaller the CV, the smaller is the spread of the data around the mean. The CV also makes possible comparisons of the spread of data around the mean of different items. For example, in the 2000 integrated survey, the CV for education is 4.55 percent and the CV for personal care products and services is 1.41 percent. Comparing the CVs for the two items, we can say that the spread of the data around the mean for education expenditure is larger than the spread of the data around the mean for personal care products and services.

## Table 1. Quintiles of income before taxes, annual means, standard errors, and coefficients of variation, Consumer Expenditure, 2000

Complete reporting of income							
All consumer units	Complete income reporters	Lowest 20 percent	Second- lowest 20 percent	Third- lowest 20 percent	Fourth- lowest 20 percent	Highest 20 percent	Incomplete income reporters
\$44,649	\$44,649	\$7,683	\$19,071	\$32,910	\$53,295	\$110,118	(1)
517.9	517.9	137.6	87.1	84.6	193.3	1613.4	(1)
1.2	1.2	1.8	0.5	0.3	0.4	1.5	(1)
\$38,041.03	\$40,234.86	\$17,939.45	\$26,547.37	\$34,713.42	\$46,791.00	\$75,093.08	\$32,059.31
336.7	356.8	399.9	622.2	412.3	626.7	850.8	713.0
0.9	0.9	2.2	2.3	1.2	1.3	1.1	2.2
\$5,157.88	5,434.76	2,673.31	4,178.21	5,183.19	6,451.56	8,679.37	4,516.56
65.8	77.4	97.0	106.8	143.6	127.7	184.0	93.3
1.3	1.4	3.6	2.6	2.8	2.0	2.1	2.1
371.81	422.87	206.4	247.93	366.12	512.9	780.2	253.67
15.6	21.2	32.7	24.7	22.3	53.7	75.4	17.9
4.2	5.02	15.9	10.0	6.1	10.5	9.7	7.1
12,318.51	12,527.38	6,508.78	8,482.33	10,857.48	14,151.75	22,610.61	11,788.92
148.8	172.1	202.2	125.9	144.9	267.4	326.7	267.7
1.2	1.4	3.1	1.5	1.3	1.9	1.5	2.3
1,852.53	2,000.22	843.47	1,298.60	1,612.83	2,261.46	3,980.12	1,500.88
38.9	54.3	61.3	72.3	86.6	79.1	198.4	62.1
2.1	2.7	7.3	5.6	5.4	3.5	5.0	4.1
7,417.36	7,567.51	3,211.97	5,042.68	7,028.41	9,223.30	13,315.32	6,985.40
101.2	110.7	133.4	222.5	234.9	211.0	322.1	211.2
1.4	1.5	4.2	4.4	3.3	2.3	2.4	3.0
2,065.67	2,120.04	1,469.87	1,987.62	1,964.07	2,312.36	2,864.12	1,919.38
30.1	31.6	71.7	63.7	50.6	61.7	64.0	60.3
1.5	1.5	4.9	3.2	2.6	2.7	2.2	3.1
1,863.50	1,957.63	836.92	1,146.62	1,609.18	2,324.39	3,866.21	1,602.97
35.6	39.1	66.4	50.7	64.0	74.7	117.7	70.3
1.9	2.0	8.0	4.4	4.0	3.2	3.0	4.4
563.62	595.33	318.28	441.98	533.59	698.91	982.91	490.56
8.0	9.8	13.1	14.8	22.7	23.0	23.0	14.3
1.4	1.7	4.1	3.4	4.3	3.3	2.4	3.0
146.47	156.11	73.04	105.17	135.61	174.92	291.47	118.38
2.2	2.3	3.2	4.3	3.9	5.9	7.0	5.0
1.5	1.5	4.4	4.1	2.9	3.4	2.4	4.3
631.93	635.52	430.25	290.47	393.09	600.05	1,461.94	625.79
28.8	35.7	59.5	41.8	48.9	53.5	107.4	56.3
4.6	5.6	13.8	14.4	12.4	8.9	7.4	9.0
318.62	333.3	257.24	316.91	366.31	390.04	335.94	275.75
8.1	11.1	13.0	18.3	19.6	16.9	17.4	10.4
2.5	3.3	5.1	5.8	5.3	4.3	5.2	3.8
775.78	831.81	364.53	594.94	832.71	1,047.19	1,318.23	619.2
19.6	22.8	50.1	53.4	55.0	67.1	55.8	49.6
2.5	2.7	13.7	9.0	6.6	6.4	4.2	8.0
1,192.44	1,344.06	332.27	1,162.95	953.01	1,217.29	3,050.11	749.99
96.8	116.2	39.0	452.9	125.6	117.0	285.9	128.1
8.1	8.6	11.7	38.9	13.2	9.6	9.4	17.1
3,364.92	4,308.33	413.14	1,250.97	2,877.80	5,424.88	11,556.55	611.86
54.7	58.2	28.0	34.0	76.5	110.3	229.4	30.5
1.6	1.4	6.8	2.7	2.7	2.0	2.0	5.0
	All consumer units \$44,649 517.9 1.2 \$38,041.03 336.7 0.9 \$5,157.88 65.8 1.3 371.81 15.6 4.2 12,318.51 148.8 1.2 1,852.53 38.9 2.1 7,417.36 101.2 1.4 2,065.67 30.1 1.5 1,863.50 35.6 1.9 563.62 8.0 1.4 146.47 2.2 1.5 631.93 28.8 4.6 318.62 8.0 1.4 146.47 2.5 5775.78 631.93 28.8 4.6 318.62 8.0 1.4 146.47 2.5 5775.78 19.6 2.5 775.78 19.6 2.5 1,192.44 96.8 8.1	All consumer units Complete income reporters   \$44,649 \$44,649   \$17.9 517.9   \$12 1.2   \$38,041.03 \$40,234.86   336.7 366.8   0.9 0.9   \$5,157.88 5,434.76   65.8 77.4   1.3 1.4   371.81 422.87   15.6 21.2   4.2 5.02   12,318.51 12,527.38   1.85 2,000.22   38.9 54.3   2.1 2.7   7,417.36 7,567.51   101.2 110.7   1.4 1.5   2,065.67 2,120.04   30.1 31.6   1.5 1.957.63   35.6 39.1   1.9 2.0   563.62 595.33   8.0 9.8   1.4 1.7   146.47 156.11   2.2 2.3   1.5 1.5	All consumer units Complete income reporters Lowest 20 percent   \$44,649 \$44,649 \$7,683   \$17.9 517.9 137.6   1.2 1.2 1.8   \$38,041.03 \$40,234.86 \$17,939.45   336.7 356.8 399.9   0.9 0.9 2.2   \$5,157.88 5,434.76 2,673.31   65.8 77.4 97.0   1.3 1.4 3.6   371.81 422.87 206.4   1.5 2.12 32.7   4.2 5.02 15.9   12,318.51 12,527.38 6,508.78   148.8 172.1 3.1   1,852.53 2,000.22 843.47   38.9 54.3 61.3   2.1 2.7 7.3   7,417.36 7,567.51 3,211.97   101.2 110.7 133.4   1.5 1.5 4.9   1,863.50 1,957.63 836.92   35.6	All consumer units Complete income reporters Lowest 20 percent Second- lowest 20 percent   \$44,649 \$44,649 \$7,683 \$19,071   \$17.9 517.9 137.6 \$7,1   \$1.2 1.2 1.8 0.5   \$38,041.03 \$40,234.86 \$17,939.45 \$26,547.37   \$36.7 356.8 399.9 2.2 2.3   \$5,157.88 5,434.76 2,673.31 4,178.21   65.8 77.4 97.0 106.8   1.3 1.4 3.6 2.6   371.81 422.87 206.4 247.93   15.6 21.2 32.7 24.7   4.2 5.02 15.9 10.0   12,318.51 12,527.38 6,508.78 8,482.33   1.2 1.4 3.1 1.5   1.852.53 2,000.22 843.47 1,298.60   38.9 54.3 61.3 72.3   1.6 7,567.51 3,211.97 5,042.68   101.2	All consumer units Complete income reporters Lowest 20 percent Second. lowest 20 percent Third- lowest 20 percent   \$44,649 \$44,649 \$7,683 \$19,071 \$32,910   \$17.9 \$17.9 1.2 1.8 0.5 0.3   \$38,041.03 \$40,234.86 \$17,939.45 \$26,547.37 \$34,713.42 412.3   \$51,75.88 5,434.76 2,673.31 4,178.21 5,183.19 106.8 143.6   1.3 1.4 3.6 2.6 2.3 1.2   \$51,157.88 5,434.76 2,673.31 4,178.21 5,183.19   1.65.8 77.4 97.0 106.8 143.6   1.4 3.6 2.6 2.8   371.81 422.87 206.4 247.93 366.12   1.2,318.51 12,527.38 6,508.78 8,482.33 10,857.48   148.8 172.7 7.3 5.6 5.4   7,417.36 7,567.51 3,211.97 5,042.68 7,028.41   101.2 110.7	Complete reporting Complete reporters Lowest 20 percent Third-lowest 20 percent Fourth-lowest 20 percent   \$44,649 \$46,649 \$7,683 \$19,071 \$32,910 \$53,295   \$1,2 1,2 1,8 \$0,5 0,3 0,4   \$38,041,03 \$40,234,86 \$17,939,45 \$26,547,37 \$34,713,42 \$46,791,00   \$33,6,7 \$0,9 0,9 2,2 2,3 1,2 1,3   \$5,157,88 \$5,437,61 \$2,673,31 4,178,21 \$18,319 6,451,66   \$6,58,77,4 \$7,0 10,6,8 \$143,8 122,7 1,3   \$15,6 21,2 32,7 24,7 22,3 53,7   \$4,2 5,02 15,9 10,0 6,41,10,15 10,5   \$14,8,81 \$12,57,38 6,508,78 8,482,33 10,857,48 14,151,75   \$14,8,81 \$172,7 7,3 5,6 5,4 3,5   \$2,000,22 843,47 \$1,298,60 1,612,83 2,261,46  38,9 5	Complete reporter Fourth lowest 20 percent Fourth lowest 20 percent Fourth lowest 20 percent Fourth lowest 20 percent Highest 20 percent   \$44,649 \$44,649 \$7,683 \$19,071 \$32,910 \$53,295 \$110,118   \$17.8 \$17.9 \$46 \$133.3 \$1613.4 \$17.3 \$46,791.00 \$75,093.08   \$33,041.03 \$40,234.66 \$17,393.45 \$26,547.37 \$34,713.42 \$46,791.00 \$75,093.08   \$356.8 \$77.4 \$70 106.8 \$13.3 \$12,77 \$184.0   \$1.3 1.4 3.6 \$2.6 \$2.8 \$2.0 \$2.1   \$371.81 422.87 206.4 \$247.93 366.12 \$512.9 780.2   \$12,318.51 \$12,527.38 6,508.78 8,482.33 10.857.48 \$14,151.75 \$22.61.46 3980.12   \$38.25.3 2,000.22 \$843.47 \$12.25.9 \$14.49 \$267.7 \$2.26   \$14.8.6 7.27 \$7.3 \$5.6 \$5.4

<sup>1</sup> Components of income and taxes are derived from complete income reporters only; see glossary.