BLS WORKING PAPERS

U.S. Department of Labor
U.S. Bureau of Labor Statistics
Office of Prices and Living Conditions

Democratic Aggregation: Issues and Implications for Consumer Price Indexes

Robert S. Martin, U.S. Bureau of Labor Statistics

Working Paper 600
July 24, 2023

Democratic Aggregation: Issues and Implications for Consumer Price Indexes

Robert S. Martin*

July 24, 2023

Abstract

This paper constructs and compares consumer price indexes (CPI) using weighting methods which differentially incorporate inflation disparities across households. Plutocratic CPIs, commonly used by statistical agencies, weight households based on their total expenditure, while democratic CPIs equally weight households to better represent average consumer experiences. I estimate democratic versions of the Bureau of Labor Statistics' CPI and Chained CPI (C-CPI) for all urban consumers using the Lowe and Tornqvist formulas, respectively. From December 2002 to June 2021, the democratic CPI-U exceeds its plutocratic counterpart by approximately 0.08 percentage points per year, on average, while the democratic C-CPI-U surpasses the plutocratic by 0.19 percentage points per year. The results indicate a negative correlation between inflation and household expenditure level over the study period. I also find weight frequency to be more important than index formula for explaining why larger differences occur for the C-CPI-U.

Keywords: Cost of living index; inflation; consumer heterogeneity

JEL Codes: C43, E31

Acknowledgments: I am grateful to David Johnson, William Johnson, Josh Klick, Sara Stanley, Anya Stockburger, Rob Cage, Brian Adams, Thesia Garner, and many others for helpful comments. All errors are my own.

This paper provides a summary of research results. The information is being released for statistical purposes, to inform interested parties, and to encourage discussion of work in progress. The paper does not represent an existing, or a forthcoming new, official BLS statistical data product or production series.

*Division of Price and Index Number Research, Bureau of Labor Statistics, 2 Massachusetts Ave, NE, Washington, DC 20212, USA. Email: Martin.Robert@bls.gov

1 Introduction

Most statistical agencies calculate consumer price indexes (CPI) using market-level expenditures and prices, corresponding to a representative consumer model. Such indexes are sometimes called "plutocratic" because they implicitly weight households by their total expenditure (Pollak, 1989; ILO, 2004). If inflation varies systematically with household expenditures, then a traditional CPI may differ from an aggregate that equally-weights households, so-called "democratic" (Prais, 1959). Plutocratic indexes may be appropriate indicators of overall macroeconomic conditions as they weight each dollar of expenditure equally. However, when constructing an indicator of consumer experiences (e.g., for adjusting payments), then equal weighting is attractive (Astin and Leyland, 2015).

This paper estimates democratic counterparts to the Bureau of Labor Statistics (BLS) CPI and Chained CPI (C-CPI) for all urban consumers. It compares plutocratic-democratic inflation gaps between the Lowe and Tornqvist formulas and for different weight frequencies. The CPI uses the Lowe formula for upper-level aggregation, while the C-CPI uses the Tornqvist (Bureau of Labor Statistics, 2023). Previous studies of democratic CPIs have tended to focus on fixed-weight indexes like the Laspeyres and Lowe using annual weights. To my knowledge, this is the first paper to analyze a household-based democratic Tornqvist for an all-items CPI. The Tornqvist or Tornqvist-like formulas have been considered previously for equally-weighted purchasing power parity indexes (Diewert, 2005), and hedonic house price indexes, such as in Hill and Scholz (2018). The building blocks for my main analysis are Lowe and Tornqvist indexes constructed for individual consumer units from the Consumer Expenditure Survey (CE). To construct consumer unit-level expenditure baskets, I modify a statistical matching method proposed by Hobijn, Mayer, et al. (2009). The democratic Lowe and Tornqvist indexes are then formed as equally-weighted averages of the respective individual indexes.

I estimate that from December 2002 to June 2021, a democratic version of the CPI-U exceeds its plutocratic counterpart by about 0.08 percentage points per year, on average, with

differences tending to decline over the sample period. The results for the Tornqvist index are more striking. Using consumer unit-based weights, I find a democratic C-CPI-U is higher than the plutocratic by an average of 0.19 percentage points per year. The pattern of higher democratic indexes is consistent with a tendency for individual inflation to co-vary negatively with household expenditure level, which matches the relationship observed between inflation and income from Jaravel (2018), Argente and Lee (2021), Kaplan and Schulhofer-Wohl (2017) and Klick and Stockburger (2021), who calculate plutocratic formulas for income quantiles. I also find differences between plutocratic and democratic indexes tend to be larger over the 2002-2008 period than over the 2009-2021 period.

The gap for the Tornqvist formula is large in price index terms. It is more than double the gap for the Lowe index and is comparable to well-known price index biases like those discussed in Moulton (2018). However, the results in Section 4 suggest much of the effect is mechanical and related to durable goods categories for which spending at the household level is likely more volatile than implicit consumption. Statistical agencies can likely avoid this issue by constructing household-level flow-of service values for big-ticket items like automobiles (such as already done with owner-occupied housing), or basing democratic measures on more aggregate pseudo-households within which average spending may closer approximate average consumption values in a given period. Coarse versions of the latter, based on quintiles of annual expenditure or annual before-tax income, result in democratic indexes which average only about 0.05 percentage points per year higher than their plutocratic counterparts. However, there are still occasional short-run differences on the order of 0.1-0.2 percentage points per year.

This paper relates to the literature on inflation heterogeneity across individuals and subpopulations such as Hobijn and Lagakos (2005) and Kaplan and Schulhofer-Wohl (2017). Jaravel (2021) gives a broad review. A review of the cost-of-living index theory as applied to heterogeneous groups can be found in Pollak (1989) and ILO (2004). Ley (2005) contains a substantial international review of price index aggregation across households, as well as

a mathematical description of how household budget differences may lead to gaps between plutocratic and democratic Lowe indexes. Democratic indexes using U.S. consumer expenditure data have been studied previously in Kokoski (2000), Hobijn, Mayer, et al. (2009), and Cage, Klick, and W. Johnson (2018), among others. These focus on earlier or more limited time periods, the Lowe formula, and low frequency weighting.

A limitation of this and many previous studies is that the BLS only samples prices to represent the urban population in a given geographic area—no available information links the price sample with individual households or groups. As a consequence, I must assume identical prices paid and expenditure shares within the item-area stratum (e.g., men's suits in Pittsburgh). Detailed demographic or household-linked expenditure microdata is not available for the full CPI basket, though many studies have examined specific categories like food and other nondurable goods. For these categories, within-stratum heterogeneity has been shown to be the major driver of inflation heterogeneity across households (Kaplan and Schulhofer-Wohl, 2017) and income deciles (Jaravel, 2018). There is indirect evidence that within-stratum inflation at the individual or group-level is also negatively correlated with expenditure. Indeed, using transactions data for consumer nondurables, Jaravel (2018) finds greater differences between top and bottom income-decile specific price indexes when they are computed using barcode-by-income level detail in prices and expenditures than when they are computed starting from more aggregated basic price indexes, which would more closely match BLS methodology. Given positive correlation between income and expenditures, I expect the results in this paper may underestimate (in terms of magnitude) for the true gaps between plutocratic and democratic indexes.

2 Plutocratic and Democratic Price Indexes

The BLS constructs price indexes in two stages. First, elementary indexes are computed for narrowly defined item-area strata, such as men's suits in Pittsburgh (Bureau of Labor

Statistics, 2023). These are then combined using either the Lowe (CPI-U) or Tornqvist (C-CPI-U) formulas based on weights from the CE. I consider adjustment to this second stage of aggregation to exploit consumer-unit level variation in the CE. This section describes the relationships between household-level expenditure data and market level price indexes. Similar expressions for the Lowe indexes, can be found in National Research Council (2002), Ley (2005), and elsewhere. For exposition purposes, I refer generically to a "household" as the basic unit chosen for aggregation in the democratic index, within which we assume joint economic decision making. I reference "consumer units" specifically as the micro units sampled in the CE. Later, I will consider the potential benefits and costs of defining a "household" as something broader than a consumer unit. For ease of exposition, I make some simplifications for notation. First, I refer to individual prices instead of elementary price indexes as the subjects of either the Lowe or Tornqvist aggregation. The formulas also omit CE sampling weights for households, but these are used in all empirical analysis. Finally, I do not explicitly incorporate geographic areas into notation, but one can consider the item indexes $i = 1, \ldots, N$ as representing item-area pairs.

Let the subscripts i and h denote elementary items and households, respectively, and let t denote a time period, usually a month. The price index compares prices in a period t against some reference period, weighting them by quantities or expenditures from a potentially different reference period. For the CPI, the price reference period is fixed (for two years) at v, while the quantity reference period is biennial and denoted b. In current practice, b ends twelve months prior to v, and both are updated every two years. For example, for t = January 2018 ... December 2019, v = December 2017 and b = 2015-2016. In the C-CPI, the index for month t has month $t-1$ as its price reference period, and the weights are derived from expenditures in both t and $t-1$.

As discussed in Section 1, we assume that all households within the same geographic region face the same price change for elementary items. Let q_{ith} represent the elementary quantity aggregate purchased at price p_{it}. Household-specific expenditures are denoted

$e_{ith} = p_{it}q_{ith}$ and expenditure shares $s_{ith} = e_{ith}/\sum_{j=1}^{N} e_{jth}$. Index formulas are traditionally written for a representative consumer, i.e., in terms of aggregate quantities $q_{it} = \sum_{h=1}^{H} q_{ith}$, expenditures $e_{it} = \sum_{h=1}^{H} e_{ith}$, and expenditure shares $s_{it} = e_{it}/\sum_{i=1}^{N} e_{it}$. Equivalently, the representative consumer's expenditure shares can be written in terms of average quantities $H^{-1}\sum_{h=1}^{H} q_{ith}$ or average expenditures $H^{-1}\sum_{h=1}^{H} e_{ith}$.

The Lowe index is an arithmetic average of price relatives p_{it}/p_{iv}. These are weighted using price-updated expenditure shares, which combine period v prices and period b quantities. For a household, we can write it as $P_{Lo,h} = \sum_{i=1}^{N} s_{i\{v,b\}h}(p_{it}/p_{iv})$, where $s_{i\{v,b\}h} = p_{iv}q_{ibh}/\sum_{j=1}^{N} p_{jv}q_{jbh}$ is the price-updated expenditure share for the household. Using the CE, we only observe consumer units for a sub-period of b (i.e., a twelve-month span). I abstract from this issue for now, but it is discussed further in Section 3. The plutocratic Lowe index is denoted P_{PLo} and written in Eq. 1. With common prices, this corresponds to the usual market-level Lowe index using market expenditure shares $s_{i\{v,b\}} = \sum_{h=1}^{H} p_{iv}q_{ibh}/\sum_{k=1}^{H}\sum_{j=1}^{N} p_{jv}q_{jbh}$. The third equality of Eq. 1 shows the well-known result that this index can be written as an expenditure-weighted (plutocratic) average of the household-specific Lowe indexes.

$$P_{PLo} = \sum_{i=1}^{N} s_{i\{v,b\}}\frac{p_{it}}{p_{iv}} = \sum_{h=1}^{H} S_{\{v,b\}h}\sum_{i=1}^{N} s_{i\{v,b\}h}\frac{p_{it}}{p_{iv}} = \sum_{h=1}^{H} S_{\{v,b\}h}P_{Lo,h} \tag{1}$$

The second equality follows from re-writing the aggregate shares as $s_{i\{v,b\}} = \sum_{h=1}^{H} S_{\{b,v\}h}s_{i\{v,b\}h}$, where $S_{\{v,b\}h} = \sum_{i=1}^{N} p_{iv}q_{ibh}/\sum_{k=1}^{H}\sum_{i=1}^{N} p_{iv}q_{ibk}$ is household h's share of total consumer expenditure. This shows how the formula gives more weight to households with higher expenditures.

The C-CPI-U, in contrast, uses the Tornqvist formula, which is a geometric mean of monthly price relatives $p_{it}/p_{i,t-1}$, weighted by the arithmetic average of the month t and $t-1$ expenditure shares. For the household, this is written $P_{T,h} = \prod_{i=1}^{N}(p_{it}/p_{i,t-1})^{w_{ith}}$, where $w_{ith} = .5\,(s_{i,t-1,h} + s_{ith})$. With common prices, the plutocratic Tornqvist can be

written a few ways, as shown in Eq. 2.

$$P_{PT} = \prod_{i=1}^{N} \left(\frac{p_{it}}{p_{i,t-1}} \right)^{w_{it}} = \prod_{h=1}^{H} \prod_{i=1}^{N} \left(\frac{p_{it}}{p_{i,t-1}} \right)^{w^*_{ith}}$$
$$= \left\{ \prod_{h=1}^{H} \left[\prod_{i=1}^{N} \left(\frac{p_{it}}{p_{i,t-1}} \right)^{s_{i,t-1,h}} \right]^{S_{t-1,h}} \right\}^{.5} \left\{ \prod_{h=1}^{H} \left[\prod_{i=1}^{N} \left(\frac{p_{it}}{p_{i,t-1}} \right)^{s_{ith}} \right]^{S_{th}} \right\}^{.5}. \quad (2)$$

The first equality shows the market level Tornqvist index using aggregate expenditure weights $w_{it} = .5\,(s_{i,t-1} + s_{it})$, while the second equality shows an aggregation across households and items using weights $w^*_{ith} = .5\,(S_{t-1,h}s_{i,t-1,h} + S_{th}s_{ith})$. This follows from the relationship $s_{it} = \sum_{h=1}^{H} S_{th}s_{ith}$, where $S_{th} = \sum_{i=1}^{N} e_{ith} / \sum_{k=1}^{H} \sum_{i=1}^{N} e_{itk}$ is household h's share of total expenditure in month t. Unlike the Lowe, the market-level Tornqvist cannot generally be written as an average of household-level Tornqvist indexes. The index can be written in terms of household price indexes, as after the third equality, but they are not Tornqvist. The terms in square brackets are household-specific geometric Laspeyres and geometric Paasche indexes, which are then averaged geometrically across h using either S_{th} or $S_{t-1,h}$ as weights.

Democratic Lowe and Tornqvist indexes are calculated simply by giving each household equal weight. For the Lowe index, we use an arithmetic average, as in Eq. 3.

$$P_{DLo} = \frac{1}{H} \sum_{h=1}^{H} \sum_{i=1}^{N} s_{i\{v,b\}h} \frac{p_{it}}{p_{iv}} = \sum_{i=1}^{N} \bar{s}_{i\{v,b\}} \frac{p_{it}}{p_{iv}}. \quad (3)$$

The last term of Eq. 3 shows the democratic Lowe looks very similar to the usual Lowe formula, but uses equally-weighted expenditure shares, given by $\bar{s}_{i\{v,b\}} = H^{-1} \sum_{h=1}^{H} s_{i\{v,b\}h}$.

I define the democratic Tornqvist as an equally-weighted geometric average of household Tornqvist indexes.

$$P_{DT} = \left(\prod_{h=1}^{H} \prod_{i=1}^{N} \left(\frac{p_{it}}{p_{i,t-1}} \right)^{w_{ith}} \right)^{1/H} = \prod_{i=1}^{N} \left(\frac{p_{it}}{p_{i,t-1}} \right)^{\bar{w}_{it}}, \quad (4)$$

where $\bar{w}_{it} = H^{-1} \sum_{h=1}^{H} w_{ith}$. The final term shows that by using a geometric mean, the demo-

cratic Tornqvist can be written similarly to the market-level Tornqvist, but using democratic weights $\bar{w}_{it}$ instead of the usual w_{it}. I use the geometric mean because it is the same type of mean used in the plutocratic formula to average across households. Jensen's inequality implies that an arithmetic average of household indexes exceeds the geometric average unless the $P_{T,h}$ are equal across h. Online Appendix A.4 shows using an arithmetic mean would increase the index by about 0.03 percentage points per year.

Index levels, or long-term changes, are formed by chaining (multiplying) shorter-term price changes. For the Lowe case, let $P(v,t)$ be either the plutocratic or democratic covering price change from v to t, as given by either Eq. 1 or Eq. 3. Then for index levels $I(t)$, we have $I(t) = I(t-1) \times [P(v,t)/P(v,t-1)]$. Similarly, we convert the monthly Tornqvist changes (Eq. 2 or Eq. 4) into index levels with the relationship $I(t) = I(t-1) \times P(t-1,t)$.

2.1 Plutocratic Gaps

Ley (2005) refers to the difference between the plutocratic and democratic index as the "plutocratic gap," written for the Lowe formula as

$$P_{PLo} - P_{DLo} = \sum_{i=1}^{N} \left(s_{i\{v,b\}} - \bar{s}_{i\{v,b\}}\right) \frac{p_{it}}{p_{iv}} = \sum_{i=1}^{N} \left(s_{i\{v,b\}} - \bar{s}_{i\{v,b\}}\right) \left(R_{it} - \bar{R}_t\right), \tag{5}$$

where $R_{it} = p_{it}/p_{iv}$ and $\bar{R}_t = N^{-1} \sum_{i=1}^{N} R_{it}$. For the Tornqvist, I define the gap in natural logs.

$$\begin{aligned} \ln P_{PT} - \ln P_{DT} &= \sum_{i=1}^{N} \left(w_{it} - \bar{w}_{it}\right) \ln \left(\frac{p_{it}}{p_{i,t-1}}\right) \\ &= \sum_{i=1}^{N} \left(w_{it} - \bar{w}_{it}\right) \left(r_{it} - \bar{r}_t\right), \end{aligned} \tag{6}$$

where $r_{it} = \ln\left(p_{it}/p_{i,t-1}\right)$ and $\bar{r}_t = N^{-1} \sum_{i=1}^{N} r_{it}$. The second equalities of Eq.'s 5 and 6 follow because the differences in weights $s_{i\{v,b\}} - \bar{s}_{i\{v,b\}}$ and $w_{it} - \bar{w}_{it}$ sum to zero and $\bar{R}_t$ and $\bar{r}_t$ are constant across i.

The plutocratic gaps equal $N-1$ times the sample covariance between the weight differences and item-level price changes, as the differences between democratic and plutocratic weights are mean zero by construction. For the Lowe index, Ley (2005) derives conditions for the plutocratic gap to exist (Online Appendix A.1 shows this is easily extended to the Tornqvist). First, there must be dispersion in expenditures across households and price change across items, which is generally true (see, e.g., Bureau of Labor Statistics (2020); Bureau of Labor Statistics (2021)). In addition, differences between plutocratic and democratic weights, $s_{i\{v,b\}} - \bar{s}_{i\{v,b\}}$, or $w_{it} - \bar{w}_{it}$, must be correlated with price changes for individual. It is not sufficient, for example, that poorer households spend a greater proportion on food than richer households. This must be accompanied by inflation rates for food items that are systematically higher or lower than average inflation.

There are two reasons to expect the Lowe and Tornqvist formulas might have different plutocratic gaps. First, the Tornqvist index approximates a cost-of-living index, while the Laspeyres is an upper bound (Diewert, 1976; Konüs, 1924). While the Lowe is technically a modified Laspeyres formula, the version in this paper uses lagged expenditure information like the Laspeyres index. A difference like $P_{PLo} - P_{PT}$ is sometimes interpreted as a "substitution effect" in that the Tornqvist formula reflects expenditure shifts in response to relative price changes while the Laspeyres and Lowe do not. Garner, D. Johnson, and Kokoski (1996) and Argente and Lee (2021) suggested lower income or impoverished households have more limited ability to substitute consumption. If so, then household-level differences $P_{Lo,h} - P_{T,h}$ may increase with total expenditure, which would correspond to a higher Tornqvist plutocratic gap.

The second reason is more mechanical and has to do with how the Tornqvist formula is used with higher-frequency weights in the C-CPI. Infrequently-purchased items can receive a lower weight in a democratic indexes even if long-run purchases are homogeneous. Further, purchases will necessarily be less frequent at a monthly frequency (the Tornqvist C-CPI) than at an annual or biennial frequency (the Lowe CPI). To understand how this occurs, suppose

there are only two items and consider their expenditure shares. In simplified notation, the expenditures at the household level are given by e_{ih}, $i = 1, 2$. The plutocratic share is

$$s_i = \frac{H^{-1} \sum_{h=1}^{H} e_{ih}}{H^{-1} \sum_{h=1}^{H} e_{1h} + H^{-1} \sum_{h=1}^{H} e_{2h}}, \; i = 1, 2 \tag{7}$$

while the democratic share is

$$\bar{s}_i = H^{-1} \sum_{h=1}^{H} \frac{e_{ih}}{e_{1h} + e_{2h}}, \; i = 1, 2. \tag{8}$$

Suppose the first item is always purchased, while the second item is purchased by the household with probability ρ. Suppose further that conditional on purchase, quantities are constant across households. This implies $e_{1h} = e_1$ for all h and $e_{2h} = d_h e_2$, where d_h is a Bernoulli random variable with mean ρ. From Slutsky's theorem and the expected value of a Bernoulli random variable, we have the following asymptotic results as H approaches infinity.

$$s_1 = \frac{e_1}{e_1 + \rho e_2} + o_p(1). \tag{9}$$

$$s_2 = \frac{\rho e_2}{e_1 + \rho e_2} + o_p(1). \tag{10}$$

$$\bar{s}_1 = \frac{e_1 + (1 - \rho) e_2}{e_1 + e_2} + o_p(1). \tag{11}$$

$$\bar{s}_2 = \frac{\rho e_2}{e_1 + e_2} + o_p(1). \tag{12}$$

Note that by construction, if $\rho = 1$, then there is no difference between the democratic and plutocratic shares. However, for $\rho < 1$, then we have (with probability approaching 1) $\bar{s}_2 < s_2$ and consequently, $\bar{s}_1 > s_1$, since shares must sum to one. Therefore, the democratic shares place lower weight on the less frequently purchased item (e.g., an automobile) while placing higher weight on the more frequently purchased item (e.g., rent). The consequences are more severe for higher frequency indexes as this lowers the probability of purchase in the

period. If consumption incidence across consumer units is linked to preferences or income, then we may want this reflected in democratic indexes. For instance, not all households buy children's clothes, pet supplies, airline tickets, or cable television services. For durable goods like automobiles, however, the pattern may be undesirable for an index of consumer prices—the absence of a household purchase does not mean the absence of consumption. Section 4 will show that weighting differences for vehicles and housing fit the pattern suggested by this analysis.

2.1.1 Pseudo-Households

Section 2 thus far has presumed individual household (or consumer unit) microdata are the basis for computing democratic weights, as in Kokoski (2000), Ley (2005) or Hobijn, Mayer, et al. (2009). Household-based indexes have a high data requirement, however. For instance, they require detailed data on the entire expenditure basket, which may not be available. Additionally, the above discussion suggests reliable durable good consumption measures at the household level are superior to raw expenditures. An alternative mentioned in ILO (2004) and elsewhere is to first group households into coarser units which are then treated as single agents. Section 2.1 suggests using groups of units may be attractive if average expenditures for durable goods within such pseudo-households approximate consumption values better than consumer-unit specific acquisition costs. The pseudo-households should be comprised of otherwise relatively homogeneous consumer units, which ensures within-group plutocratic gaps can be ignored. I choose a few relatively coarse groupings to explore as pseudo-household definitions for democratic indexes in Section 4: quintiles of monthly expenditure, quintiles of annual expenditure, and quintiles of annual before-tax income. These groupings are coarser than the expenditure share-based clusters used by Cage, Klick, and W. Johnson (2018), who are motivated by measurement error and nonhomotheticity rather than infrequent purchases. Applying their procedure to a monthly-weighted index would be problematic because the group selection would be sensitive to large, infrequent purchases.

To explain why more homogeneous household groupings are preferred, I decompose the Lowe plutocratic gap into between-group and within-group components, with the groups corresponding pseudo-households. Because the Tornqvist index is approximately consistent in aggregation (Diewert, 1978), the intuition carries over even if the exact expressions do not. Suppose we partition the H households into G groups indexed by g. Let $S_{P,g}$ be group g's share of total expenditure and $S_{D,g}$ its share of the total population. We can then write the plutocratic gap as:[1]

$$P_{PLo} - P_{DLo} = \sum_{g=1}^{G} S_{P,g} P_{PLo,g} - \sum_{g=1}^{G} S_{D,g} P_{DLo,g} \tag{13}$$

$$= \sum_{g=1}^{G} \left(S_{P,g} - S_{D,g}\right) \bar{P}_g + \sum_{g=1}^{G} \bar{S}_g \left(P_{PLo,g} - P_{DLo,g}\right), \tag{14}$$

where $\bar{P}_g = .5\left(P_{PLo,g} + P_{DLo,g}\right)$, $\bar{S}_g = .5\left(S_{P,g} + S_{D,g}\right)$, and $P_{PLo,g}$ and $P_{DLo,g}$ are the plutocratic and democratic Lowe indexes, respectively, computed over households in group g.

The first term of Eq. 14 is the between-group component. It captures differences in inflation and expenditures across groups and is the plutocratic gap that would prevail if within-group spending patterns were either homogeneous or uncorrelated with price changes. The second term of Eq. 14 is the within-group component. It captures the differences in inflation within the group and is the plutocratic gap that would prevail if average inflation was the same across groups or if group-level spending was proportional to its share of the population. A democratic index based on pseudo-households is necessarily formed by expenditure-weighting households in the first stage. Therefore, it makes sense to choose groupings which would minimize the "within" component of the plutocratic gap. One way to accomplish this would be to choose groupings which would minimize within-group variance in household consumption if properly measured with flow values for durable goods.

One potential method that I explore in Section 4 is to use quantiles of the total ex-

[1]This follows Eq. 1 of Jaravel (2018), who similarly decomposes the rich-poor inflation gap into between-stratum and within-stratum components.

penditure distribution even though they reflect lump-sum purchases of durables. Ideally the period for expenditures should be long enough so that group membership is not overly sensitive to a single large purchase. Greater homogeneity in all shares will also reduce the within-group gap. To use a clustering method like in Cage, Klick, and W. Johnson (2018) for the Tornqvist index, however, one may wish to exclude durable goods categories from the similarity measures, as well as match on month $t-1$ and t expenditure shares concurrently.

3 Data and Processing

This section briefly describes the data sources used in calculation of the CPI and highlight relevant features for constructing household-weighted measures. More complete information is available in Bureau of Labor Statistics (2022) and Bureau of Labor Statistics (2023). I also briefly describe how I link CE Diary and Interview observations and process the data for consumer unit-level price indexes.

3.1 CPI Elementary Item-Area Indexes

Since 2018, the lowest level of aggregation in the CPI consists of 243 items and 32 geographic areas for a total of 7,776 elementary indexes, with similar dimensions in earlier periods. Within each item-area, individual price quotes are sampled to be representative of the products purchased and outlets patronized by urban consumers. These are then aggregated using either a modified Laspeyres or geometric mean formula, depending on the product category. The same elementary indexes are the basic inputs to both the CPI and C-CPI. The CPI undergoes periodic changes in the item and area samples. To simplify processing, I create a constant item classification structure over time by combining a few elementary aggregates. I also bound the monthly item-area index relatives to be between 0.05 and 20, which amounts to censoring a few monthly price changes.

3.2 Consumer Expenditure Survey

The CE is a nationwide survey designed to measure the full range of spending for the noninstitutionalized civilian population. The survey also collects information on income and demographic characteristics. The sample unit is known as a consumer unit (CU). While often corresponding to a household, a CU is a group of individuals who make joint expenditure decisions (Bureau of Labor Statistics, 2022).

The CE consists of two separate surveys—the Interview and the Diary—comprising different samples and categories of expenditure. The Interview survey has a three month recall period and is designed to cover major purchases (e.g., major appliances) and recurring items (e.g., rent, utilities). Consumer units are interviewed once every three months for four consecutive quarters on a rolling basis. The reference period for expenditures for each interview is the prior three calendar months. Roughly 7,000 consumer units complete the interview in a given quarter, with approximately 25% of the sample turning over each quarter. In contrast, the Diary consists of both a smaller sample and shorter recall period. Approximately 1,700 consumer units provide usable diaries in a given quarter, recording expenditures for two consecutive one-week periods. The survey covers more minor and frequent expenditures, such as food and apparel. The CE treats separate Interview and Diary waves as independent. I do the same for the weekly diaries, but I link interviews for the same consumer unit across collection quarters. The latter allows for a larger sample when constructing Tornqvist indexes because I can use adjacent months a consumer unit's expenditure even if they were collected in different quarters. It also allows me to construct annual expenditures for the subset of consumer units who completed four interviews. For my analysis, I use the CE sampling weights, FINLWT21, which technically treat Interviews as independent.

Table 1 gives the share of CPI-covered expenditure by major category and survey source. The underlying aggregations incorporate sampling weights and adjustments made by CPI for recall period differences across the two surveys. Roughly 75% of expenditure covered by the CPI is sourced from the Interview, while the remaining 25% of CPI expenditures

are sourced from the Diary. The Interview provides the vast majority of expenditures for Housing, Transportation, Medical Care, and Education and Communication, while the Diary is the predominant source for Food and Beverages and Apparel. Significant proportions of Recreation and Other Goods and Services come from each survey. [Insert Table 1 near here.]

The Interview survey is the natural starting point for constructing household-based measures of inflation, as participation lasts up to one year (versus two weeks for the diary) and coverage includes most CPI-eligible expenditure categories. I use all available consumer units for which I can compute individual Tornqvist indexes. As these depend on current and prior month expenditures, I can not compute them until a consumer unit's second month in the sample. Since I base the Lowe indexes on the same biennial reference periods as the published CPI, I use a separate sample which is restricted to consumer units who completed all four interviews and have four complete quarters of spending within the biennial reference period. For the 2013-2014 reference period, for example, I take consumer units who participated in four interviews and had complete reference quarters occurring between January 2013 and December 2014. I sum their expenditures by item, and then the consumer unit-item level expenditures are price-updated to the pivot month, December 2015. These shares are combined with elementary index relatives based to December 2015 and ending between January 2016 and December 2017. While all consumer unit-level shares are drawn from within the same biennial reference period, the shares themselves are necessarily at an annual frequency. Requiring four quarters from each CU reduces the variation in total expenditures at the consumer unit level stemming from either normal sample rotation or attrition. Variation in total expenditures across households is a determinant of the plutocratic gap (Ley, 2005), particularly if tied to infrequent purchases as shown in Section 2.1.In the online appendix, Figure A8 shows the impact of consumer unit selection is quite small for the plutocratic Lowe and Tornqvist indexes.

For those categories collected in the Interview, I generally assume a CU not reporting values for an item had expenditures of zero, though as discussed, differences between ex-

penditures and consumption may be empirically relevant for big-ticket items. Expenditures on categories usually collected in the Diary, however, are imputed using an unconstrained statistical match based on Hobijn, Mayer, et al. (2009) and described further in Online Appendix A.2. A donor Diary is selected for each Interview based on a model of predicted total expenditure as a function of demographic characteristics. Subsequent index calculation in Section 4 uses the synthetic consumer unit observations for both plutocratic and democratic aggregates, in order to control for the effect of the matching process and isolate the effect of equally-weighting households. Online Appendix A.4 shows that CPI-U and C-CPI-U replications using the matched data closely replicate their published counterparts.

I also set negative expenditures at the CU-item-month level to zero, so that the weights for individual indexes are all weakly positive. Negative expenditures are occasionally recorded for certain items, like medical care, for which the consumer unit receives reimbursement. Positive expenditure shares at the household level seems a reasonable requirement for a democratic index. Nevertheless, while Figure A8 in the online appendix suggests that bounding CU-level expenditures to be positive has a small negative effect on average, in the earliest years of the sample, it lowers the Plutocratic Lowe by about a tenth of a percentage point per year on average. For this reason I base results on plutocratic and democratic indexes that both use the same bounding rule. Finally, the CE survey had area sample revision in January 2015 which included a change in consumer unit identifiers. As a Tornqvist index requires expenditures in adjacent months, I match 1,315 CUs from continuing index areas with their previous identifiers by matching income, core-based statistical area, age, housing tenure, race, sex, occupation, and family size. As a result, sample of CUs for which I can compute a Tornqvist index in January 2015 is small, only about 25% of the sample for February 2015. Nevertheless, there is no structural break in the timeseries for this period.

Finally, I also omit two weight-smoothing procedures used in the CPI and C-CPI—composite estimation for the biennial item-area Lowe weights and ratio-allocation for the monthly item-area Tornqvist weights. Both of these are designed to lower the sampling

variance of weights for specific item-areas. Figure A8 in the online appendix shows my national-level CPI-U and C-CPI-U replications closely match the published indexes even without these procedures.

4 Results

4.1 Indexes Based on Consumer Units

As discussed previously, necessary conditions for plutocratic gaps include expenditure and inflation inequality across consumer units, as well as differences in spending patterns across the expenditure distribution. Online Appendix A.3 documents such heterogeneity in the CE data, with findings qualitatively similar to Kaplan and Schulhofer-Wohl (2017) and others.

Table 2 and Figure 1 show plutocratic gaps tend to be positive for both the Lowe and Tornqvist index formulas, but that the average Tornqvist gap is about two and a half times the magnitude of the average Lowe gap. Over the period December 2002 to June 2021, the average twelve month percent change of the democratic Lowe exceeds that of the plutocratic Lowe by 0.08 percentage points per year. The Lowe findings are roughly consistent with earlier findings by Kokoski (2000) and Hobijn, Mayer, et al. (2009). Kokoski (2000) finds from 1987-1997, a democratic modified Laspeyres index exceeds the plutocratic version by 0.5 percentage points cumulative, or 0.05 percentage points per year. Looking at 1984-2004, Hobijn, Mayer, et al. (2009) find the difference between a democratic and plutocratic Laspeyres-type indexes to be "less than 3 percentage points" cumulative, or less than 0.14 percentage points per year. For the 2002-2021 period, however, however, the democratic Tornqvist exceeds the plutocratic Tornqvist by a larger magnitude, 0.19 percentage points per year on average. Gaps in monthly and twelve-month percent change vary over time both in sign and magnitude. Table 2 implies that the plutocratic gaps from 2002 to 2008 tend to be larger in magnitude than those in later periods. In 2008, for instance, the Lowe gap averages -0.33 percentage points while the Tornqvist averages -0.61 percentage points.

From 2010 to 2021, average gaps in the Lowe indexes tend to be relatively small and are both positive and negative. Similarly, the Tornqvist gaps also tend to be smaller in magnitude over this time frame, but larger average gaps still occur in 2017 and 2018, and 2021 (January to June), where the gap is also positive. While the signs of the gaps are not consistent over time, in the long run, the democratic index levels tend to be higher than the plutocratic, as shown in Figure 1. In particular, the Tornqvist gap is substantial in price index terms, as it is similar in magnitude to the difference between the Lowe CPI-U and the Tornqvist C-CPI-U. In online appendix A.4, I show the plutocratic Lowe and Tornqvist I create from consumer unit indexes track the published CPI-U and C-CPI-U well. [Insert Table 2 near here.] [Insert Figure 1 near here.]

Whether the larger Tornqvist gap makes economic sense is the focus of the rest of this section. I find similar qualitative patterns if indexes are computed using only the CE Interview with aggregate food spending, but the plutocratic indexes do not track the CPI-U and C-CPI-U as well. I also find using an arithmetic mean to compute the democratic Tornqvist results in a slightly larger plutocratic gap, on average -0.22 percentage points per year instead of -0.19. Finally, I use a few simple equivalence scales based on family size when calculating of the democratic Tornqvist and find these make relatively little difference. [Insert Figure 2 near here.]

Demand theory may give reason to expect a more negative plutocratic gap for the Tornqvist than for the Lowe. Such a pattern is consistent with higher expenditure households having a greater ability to substitute away from items with higher relative price change, similar to findings in Garner, D. Johnson, and Kokoski (1996). The Tornqvist index approximates a theoretical cost-of-living index, which reflects such substitutions, while the fixed-weight Lowe index does not when based on lagged expenditures. By weighting households equally, the democratic Tornqvist index may reflect less substitution, on average, which would have a tendency to push it higher than the plutocratic Tornqvist. To assess this explanation, I compare Lowe and Tornqvist indexes for different segments of the expenditure

distribution. I restrict attention to consumer units which completed four Interviews, for which we observe roughly one year of spending. For the weight reference sample (for the Lowe) or by month (for the Tornqvist), I divide consumer units who supply indexes into quintiles based on annual expenditure. I use annual expenditures for both so that the quintiles are more comparable, as monthly expenditures are more volatile and less equal. Figure 2 shows the average differences between the Lowe and Tornqvist twelve month percent changes by quintile. These average differences are interpreted as the degree to which each quintile substitutes away from items with higher relative price change. Figure 2 suggests an inverse, though nonmonotonic, relationship between expenditure quintile and substitution effect, the opposite of substitution patterns which would explain the differential plutocratic gaps. For instance, weighted equally, household Lowe indexes for the first quintile average about 0.14 percentage points per year higher than the household Tornqvist indexes. The margin is 0.18 percentage points for the second quintile, 0.15 percentage points for the third quintile, 0.09 percentage point for the fourth quintile, and 0.02 percentage points for the fifth quintile.

Instead, the tendency for the Tornqvist plutocratic gaps to be larger than the Lowe stems from the higher frequency of the weights. Figure 3 plots the difference in twelve month percent changes between the plutocratic and democratic index for the Lowe and Tornqvist formulas respectively. The figure presents the same Lowe and Tornqvist indexes as in Table 2, as well as variants of the Lowe index methodology where the household is defined differently, either as a consumer unit-quarter combination or consumer-unit month combination, still within the same (lagged) biennial weight reference period as the baseline Lowe which uses annual expenditures at the consumer unit level. These variants have the effect of making the democratic Lowe indexes based on higher frequency expenditure shares, but have no or little effect on the plutocratic Lowe indexes. For the Lowe indexes based on monthly or quarterly shares, I also use the full sample of consumer units, though this has very little effect on the indexes or plutocratic gaps. As Figure 3 shows, increasing the weight frequency for the Lowe index increases the magnitude and volatility of the plutocratic gaps,

making them track much closer to those of the Tornqvist index. [Insert Figure 3 near here.]

Plutocratic gaps increase with the dispersion in expenditures across households (Ley, 2005). If monthly expenditures are more dispersed than annual expenditures, one might expect there to be larger plutocratic gaps for the monthly chained Tornqvist formula than the biennial Lowe. Nevertheless, the compositions of the plutocratic gaps suggest that rather than general dispersion, there are mechanical differences between the plutocratic and democratic weights (particularly for the Tornqvist indexes) that match the theory from Section 2.1 which suggests at higher frequencies, infrequent purchases like automobiles tend to receive lower weight and regular purchases like rent tend to receive higher weight in a democratic index.

Indeed, the majority of differences across formulas appear driven by transportation and housing. Figure 4 decomposes the plutocratic gaps for the 2018-19 period (see Eq.'s 5 and 6) into the contributions by major group. The Lowe plutocratic gap is expressed in level differences, while the Tornqvist plutocratic gap is expressed in natural log differences, but the magnitudes are small enough to be roughly comparable. By far, the two largest contributors to the Tornqvist plutocratic gap are transportation and housing, both which have a negative contribution. Figure 5 further examines the weight differences and relative inflation levels by major group. This figure ignores within major group correlations between share differences and price changes which also contribute to the plutocratic gap, but is suggestive of the overall effects shown in Figure 4. For instance, from Figure 4, we see that food contributes relatively little to the plutocratic gaps. Panel (a) of Figure 5 shows that while both the Lowe and Tornqvist plutocratic weights for food are substantially lower than the democratic weights, panel (b) shows food inflation differs little from average inflation during this time period. Similarly, inflation for apparel is lower than average inflation during this period, but weight differences are minimal between the plutocratic and democratic indexes. For both index formulas, the plutocratic index places a lower weight on housing and a higher weight on Transportation, but the differences are much larger for the Tornqvist than for the Lowe.

This combined with higher-than-average inflation for these categories drives the different plutocratic gaps. [Insert Figure 4 near here.][Insert Figure 5 near here.]

Figures 6 and 7 repeat Figures 4 and 5 for the five most significant item strata in terms of contributing to the differences between the Lowe and Tornqvist plutocratic gaps. Indeed, we see that rent and owner equivalent rent (for which consumer units are more likely to have have expenditures each month), as well as new vehicles, used vehicles and airline fares (which are less frequently purchased) are the largest contributors. In the case of rent and owner equivalent rent, the higher frequency of the Tornqvist index increases the democratic weights. For owner equivalent rent, this leads to a positive, but smaller weight difference relative to the Lowe, while for rent, it leads to a more negative weight difference relative to the Lowe. For vehicles and airline fares, we see larger negative weight gaps for the Tornqvist than we do the Lowe, reflecting even lower weight in the democratic Tornqvist index due to infrequency of purchase. The simple example from Section 2.1 suggests that the infrequent purchase of vehicles may drive the weight differences for rents as well as vehicles. Together, the two rent and two vehicle items' contributions to the Tornqvist plutocratic gap sum to -0.0036 log points for the 2018-19 period, slightly more than the total plutocratic gap in logs, which is -0.0035. It is probable that a consumption measure for vehicles would lead to smaller weight differences, and therefore a smaller plutocratic gap, due to less volatility relative to an acquisitions measure. [Insert Figure 6 near here.] [Insert Figure 7 near here.]

Figure 8 shows that in terms of the overall plutocratic gaps, rent and vehicles were also the largest contributors for the 2002-03 period as well as the 2018-19 period. The 2002-03 period is interesting because both the Lowe and Tornqvist indexes have substantial plutocratic gaps, though the Tornqvist is still much larger (-1.4 percentage points cumulative for the Tornqvist versus -0.4 percentage points cumulative for the Lowe). The impact of vehicles on weight differences is even larger in 2002-03 than it is in 2018-19. In the earlier period, gasoline is also a significant contributor to the negative plutocratic gap, as speculated by Hobijn, Mayer, et al. (2009) for the 2000-2004 period, but its contribution is overshadowed by automobiles

and rent, even for the Lowe index. [Insert Figure 8 near here.]

A consumption measure for vehicles at the consumer unit level is beyond the scope of this paper, but the following subsection explores how this issue may be less prevalent if the "household" is defined as something coarser than a consumer unit.

4.2 Indexes Based on Pseudo-Households

Section 2.1.1 discussed how democratic indexes based on coarser pseudo-households may be preferred to indexes based on consumer units from the standpoint of smoothing durable goods expenditures, provided within-group plutocratic gaps are negligible. Table 3 compares the plutocratic gaps for the different household definitions, including the CU-based plutocratic Lowe and Tornqvist from Table 2 for reference. The alternative household definitions in this table are quintiles of annual expenditure and quintiles of annual before-tax income. In both the Lowe and the Tornqvist case, democratic indexes based on coarser household definitions are lower than those based on consumer units, leading to smaller magnitude plutocratic gaps. For the Lowe, the plutocratic gap using quintiles of annual expenditure is -0.07 percentage points on average, while it is -0.04 on average using quintiles of annual before-tax income, versus -0.08 on average when based on consumer units. For the Tornqvist, the reductions in the plutocratic gap are more substantial. Using quintiles of annual expenditure reduces the gaps to -0.04 percentage points on average, and using quintiles of annual income reduces them to -0.05 percentage points on average, versus -0.19 when basing the indexes on consumer units. Notably, the Lowe and Tornqvist plutocratic gaps are much closer in magnitude using coarser pseudo-households than when using consumer unit-level indexes. [Insert Table 3 near here.]

Expenditure-based definitions may be preferred to income-based definitions for reasons discussed in Section 2.1.1. As income is an imperfect correlate with expenditure, within-group plutocratic gaps may still be present when the groups are based on income. Figure 9 shows that using the coarser household definitions (even that based on income) indeed

reduces the impact of housing and transporation weight differences in the Tornqvist case. The Tornqvist weight differences for housing are still negative, and the weight differences for transportation are still positive, but of a much lower magnitude. In addition, they are much closer to the Lowe weight differences which are based on consumer unit weights. Notably, if we choose quintiles of monthly expenditure as the pseudo-households, the housing and transportation weight differences are still quite substantial, as these groupings are sensitive to infrequent, big-ticket purchases. For instance, in 2019, consumer units that purchased a new or used vehicle had monthly expenditure percentiles that were 29.4 points higher, on average, than consumer units without vehicle purchases. [Insert Figure 9 near here.]

5 Conclusion

A democratic consumer price index may serve as an additional inflation indicator of interest to researchers and policymakers, as it tracks the average price change across households rather than across dollars of expenditure in the economy. For example, a household-weighted index for low-income populations may be useful for poverty analysis. This paper shows when using the Lowe or the Tornqvist formula, such an index tends to imply higher inflation on average than the traditional plutocratic aggregation. For several years between 2002 and 2008, the plutocratic gaps are of magnitudes on par with some of the well-known biases discussed in price index literature (Moulton, 2018), but in the long-run, average less than 0.1 percentage points in magnitude for the Lowe index. A democratic Tornqvist index based on monthly consumer unit weights implies a much larger plutocratic gap on average, but this is largely due to infrequent durable goods purchases like automobiles, which at the micro level do not reflect consumption in an economic sense. Using coarser pseudo-household definitions, I find Tornqvist plutocratic gaps to be much smaller, on the order of the Lowe plutocratic gaps. While the later period of my analysis (2010 to 2021) featured smaller plutocratic gaps, the larger gaps in the earlier period suggest that plutocratic indexes are not always

a sufficient proxy for democratic indexes. In addition, results from Jaravel (2018) suggest using common item-area indexes may understate inflation differences between households across different points in the income distribution. If so, then plutocratic gaps may be larger when starting from less aggregated price and expenditure data.

There are several interesting avenues for future research. To begin with, it would be preferable to use flow-of-service approaches at the consumer unit level for all big-ticket items, such as rental equivalence or user cost, as these purchases can have an out-sized effect on democratic weights. In addition, while this paper studies democratic aggregation with indexes that replicate many features of BLS methodology, analogs to procedures like weight-smoothing and variance estimation should be explored. Pseudo-household methods could also be implementable without matching or with more limited imputation. For instance, by only imputing total annual expenditure for the Diary and Interview samples, quintile-based democratic indexes would be calculable using methods similar to the income quartile indexes computed by Klick and Stockburger (2021). Finally, one may raise again the question of "whose index?" as posed previously in Prais (1959) and Ley (2005). The plutocratic gaps described in this paper are small relative to the overall distribution of household inflation as documented in Hobijn, Mayer, et al. (2009) and Kaplan and Schulhofer-Wohl (2017), or even the differences by income as presented in Klick and Stockburger (2021). Depending on the objectives of policymakers, measurements of inflation dispersion may be of interest along with inflation averages for a diverse set of subpopulations.

References

Argente, David and Munseob Lee (2021). "Cost of living inequality during the great recession". In: *Journal of the European Economic Association* 19.2, pp. 913–952. DOI: 10.1093/jeea/jvaa018.

Astin, John and J Leyland (2015). "Towards a household inflation index". Report commissioned by the Royal Statistical Society. URL: https://rss.org.uk/RSS/media/News-and-publications/Publications/Reports%20and%20guides/Astin-Leyland-HII-paper-Apr-2015.pdf.

Bureau of Labor Statistics (2020). *Table R-1. All consumer units: Annual detailed expenditure means, standard errors, coefficients of variation, and weekly (D) or quarterly (I) percents reporting, Consumer Expenditure Survey, 2019.* Tech. rep. URL: https://www.bls.gov/cex/tables/calendar-year/mean/cu-all-detail-2019.pdf.

— (2021). *Table 2. Consumer Price Index for All Urban Consumers (CPI-U): U. S. city average, by detailed expenditure category, August 2021.* Tech. rep. URL: https://www.bls.gov/news.release/cpi.t02.htm.

— (2022). "Consumer Expenditures and Income". In: *Handbook of Methods*. Bureau of Labor Statistics. URL: https://www.bls.gov/opub/hom/cex/home.htm.

— (2023). "The Consumer Price Index". In: *Handbook of Methods*. Bureau of Labor Statistics. URL: https://www.bls.gov/opub/hom/cpi/home.htm.

Cage, Robert A., Thesia Garner, and Javier Ruiz-Castillo (2002). "Constructing Household Specific Consumer Price Indexes: An Analysis of Different Techniques and Methods". BLS Working Paper 354. URL: https://www.bls.gov/osmr/research-papers/2002/pdf/ec020030.pdf.

Cage, Robert A., Joshua Klick, and William Johnson (2018). "Population Subgroup Price Indexes: Evidence of Heterogeneity or Measurement Error?" Meeting of the Group of Experts on Consumer Price Indexes, United Nations Economic Commission for Europe Geneva, Switzerland, May 7-9. URL: https://www.unece.org/fileadmin/DAM/stats/documents/ece/ces/ge.22/2018/United_States.pdf.

Diewert, W. Erwin (1976). "Exact and superlative index numbers". In: *Journal of Econometrics* 4.2, pp. 115–145. DOI: 10.1016/0304-4076(76)90009-9.

Diewert, W. Erwin (1978). "Superlative index numbers and consistency in aggregation". In: *Econometrica* 46.4, pp. 883–900. DOI: 10.2307/1909755.

— (2005). "Weighted country product dummy variable regressions and index number formulae". In: *Review of Income and Wealth* 51.4, pp. 561–570. DOI: 10.1111/j.1475-4991.2005.00168.x.

Garner, Thesia, David Johnson, and Mary Kokoski (1996). "An experimental consumer price index for the poor". In: *Monthly Labor Review* 119, p. 32. URL: https://www.bls.gov/opub/mlr/1996/09/art5full.pdf.

Hill, Robert J. and Michael Scholz (2018). "Can geospatial data improve house price indexes? A hedonic imputation approach with splines". In: *Review of Income and Wealth* 64.4, pp. 737–756. DOI: 10.1111/roiw.12303.

Hobijn, Bart and David Lagakos (2005). "Inflation inequality in the United States". In: *Review of Income and Wealth* 51.4, pp. 581–606. DOI: 10.1111/j.1475-4991.2005.00170.x.

Hobijn, Bart, Kristin Mayer, Carter Stennis, and Giorgio Topa (2009). "Household inflation experiences in the US: a comprehensive approach". Federal Reserve Bank of San Francisco Working Paper 2009-19. URL: http://www.frbsf.org/publications/economics/papers/2009/wp09-19bk.pdf.

Houthakker, Hendrik S. (1957). "An international comparison of household expenditure patterns, commemorating the centenary of Engel's law". In: *Econometrica*, pp. 532–551.

ILO (2004). *Consumer Price Index Manual: Theory and Practice*. Ed. by Peter Hill. Jointly published by ILO, IMF, OECD, UNECE, Eurostat, and the World Bank. URL: https://www.ilo.org/global/statistics-and-databases/WCMS_331153/lang--en/index.htm.

Jaravel, Xavier (2018). "The Unequal Gains from Product Innovations: Evidence from the U.S. Retail Sector". In: *The Quarterly Journal of Economics* 134.2, pp. 715–783. DOI: 10.1093/qje/qjy031.

Jaravel, Xavier (2021). "Inflation Inequality: Measurement, Causes, and Policy Implications". In: *Annual Review of Economics* 13. DOI: 10.1146/annurev-economics-091520-082042.

Kaplan, Greg and Sam Schulhofer-Wohl (2017). "Inflation at the household level". In: *Journal of Monetary Economics* 91, pp. 19–38. DOI: 10.1016/j.jmoneco.2017.08.002.

Klick, Joshua and Anya Stockburger (2021). "Experimental CPI for lower and higher income households". BLS Working Paper 537. URL: https://www.bls.gov/osmr/research-papers/2021/pdf/ec210030.pdf.

Kokoski, Mary (2000). "Alternative CPI aggregations: two approaches". In: *Monthly Labor Review* 123, p. 31. URL: https://www.bls.gov/osmr/research-papers/2003/pdf/ec030070.pdf.

Konüs, Alexander A. (1924). "The problem of the true index of the cost of living". In: translated in *Econometrica* (1939) 7, pp. 10-29.

Ley, Eduardo (2005). "Whose inflation? A characterization of the CPI plutocratic gap". In: *Oxford Economic Papers* 57.4, pp. 634–646.

Moulton, Brent R. (2018). "The Measurement of Output, Prices, and Productivity: What's Changed Since the Boskin Commission?" Brookings Institution Hutchins Center Report. URL: https://www.brookings.edu/wp-content/uploads/2018/07/Moulton-report-v2.pdf.

National Research Council (2002). *At What Price?: Conceptualizing and Measuring Cost-of-Living and Price Indexes*. Ed. by Charles Schultze and Christopher Mackie. National Academies Press.

Pollak, Robert A. (1989). *The Theory of the Cost-of-Living Index*. Oxford University Press on Demand.

Prais, Sigbert J (1959). "Whose cost of living?" In: *the Review of Economic Studies* 26.2, pp. 126–134. DOI: 10.2307/2296170.

Webber, Dominic and Richard Tonkin (2013). *Statistical matching of EU-SILC and the Household Budget Survey to compare poverty estimates using income, expenditures and material deprivation*. Eurostat-Methodologies and Working Papers, Luxembourg: Publications Office. URL: https://ec.europa.eu/eurostat/documents/3888793/5857145/KS-RA-13-007-EN.PDF/37d4ffcc-e9fc-42bc-8d4f-fc89c65ff6b1.

Tables

Table 1: Shares of CPI-Eligible Expenditure by Source, 2019

Major Group	Diary	Interview	Total
Food and Bev.	0.1441	0.0026	0.1468
Housing	0.0233	0.3920	0.4153
Apparel	0.0251	0.0044	0.0295
Transportation	0.0010	0.1666	0.1676
Medical Care	0.0045	0.0831	0.0877
Recreation	0.0220	0.0337	0.0558
Educ. and Comm.	0.0027	0.0659	0.0685
Other	0.0099	0.0190	0.0289
Total	0.2327	0.7673	1.0000

Note: the CPI definition of "Housing" expenditure includes owner equivalent rent.

Table 2: Consumer Unit Indexes: Average 12 Month % Change, December 2002 - June 2021

	P. Lowe	D. Lowe	P. Torn.	D. Torn.	Lowe Gap	Torn. Gap
2003	2.07	2.26	1.81	2.48	−0.19	−0.67
2004	2.48	2.65	2.28	2.73	−0.17	−0.45
2005	3.29	3.36	2.94	3.26	−0.08	−0.33
2006	3.18	3.27	2.89	3.23	−0.09	−0.34
2007	2.77	2.95	2.44	2.97	−0.18	−0.53
2008	3.80	4.13	3.80	4.41	−0.33	−0.61
2009	−0.39	−0.26	−0.36	−0.44	−0.13	0.08
2010	1.68	1.75	1.59	1.49	−0.07	0.10
2011	3.16	3.25	2.99	3.14	−0.09	−0.15
2012	2.05	2.10	1.94	1.95	−0.05	−0.01
2013	1.45	1.47	1.25	1.30	−0.02	−0.05
2014	1.62	1.68	1.56	1.66	−0.06	−0.09
2015	0.16	0.08	0.01	−0.22	0.08	0.23
2016	1.26	1.20	0.95	0.97	0.06	−0.02
2017	2.16	2.22	1.67	1.98	−0.06	−0.31
2018	2.43	2.45	1.85	2.13	−0.02	−0.28
2019	1.80	1.74	1.48	1.54	0.06	−0.06
2020	1.23	1.28	1.01	1.10	−0.04	−0.09
2021	3.22	3.27	3.20	2.96	−0.05	0.24
Average	2.04	2.12	1.82	2.01	−0.08	−0.19

Note: 2021 averages cover January-June.

Table 3: Gaps by Household Definition, December 2002 - June 2021 (Average 12 m. % ch.)

	Lowe Plutocratic Gaps				Tornqvist Plutocratic Gaps			
	P. Lowe	CU	QAE	QAI	P. Torn	CU	QAE	QAI
2003	2.07	−0.19	−0.19	−0.03	1.81	−0.67	−0.20	−0.03
2004	2.48	−0.17	−0.17	−0.03	2.28	−0.45	−0.06	−0.05
2005	3.29	−0.08	−0.09	−0.03	2.94	−0.33	−0.07	−0.08
2006	3.18	−0.09	−0.09	−0.01	2.89	−0.34	−0.05	−0.04
2007	2.77	−0.18	−0.17	−0.06	2.44	−0.53	−0.18	−0.12
2008	3.80	−0.33	−0.31	−0.11	3.80	−0.61	−0.24	−0.12
2009	−0.39	−0.13	−0.10	−0.06	−0.36	0.08	0.00	−0.05
2010	1.68	−0.07	−0.06	−0.04	1.59	0.10	0.02	−0.03
2011	3.16	−0.09	−0.09	−0.03	2.99	−0.15	−0.04	−0.07
2012	2.05	−0.05	−0.04	−0.03	1.94	−0.01	0.01	−0.04
2013	1.45	−0.02	−0.02	−0.02	1.25	−0.05	−0.02	−0.02
2014	1.62	−0.06	−0.07	−0.05	1.56	−0.09	−0.06	−0.05
2015	0.16	0.08	0.07	−0.02	0.01	0.23	0.01	−0.02
2016	1.26	0.06	0.05	0.00	0.95	−0.02	0.03	0.02
2017	2.16	−0.06	−0.07	−0.04	1.67	−0.31	−0.09	−0.05
2018	2.43	−0.02	−0.02	−0.02	1.85	−0.28	0.07	−0.05
2019	1.80	0.06	0.05	0.00	1.48	−0.06	0.08	−0.01
2020	1.23	−0.04	−0.04	−0.07	1.01	−0.09	−0.08	−0.01
2021	3.22	−0.05	−0.06	−0.03	3.20	0.24	0.18	0.01
Average	2.04	−0.08	−0.07	−0.04	1.82	−0.19	−0.04	−0.05

Note: 2021 averages cover January-June. CU means the base unit for the democratic index is the consumer unit. QAE means the base unit is the quintile of annual expenditure. QAI means the base unit is the quintile of annual before-tax income.

Figures

Figure 1: Consumer Unit Based Plutocratic and Democratic Price Indexes (Dec. 2001 = 1.0)

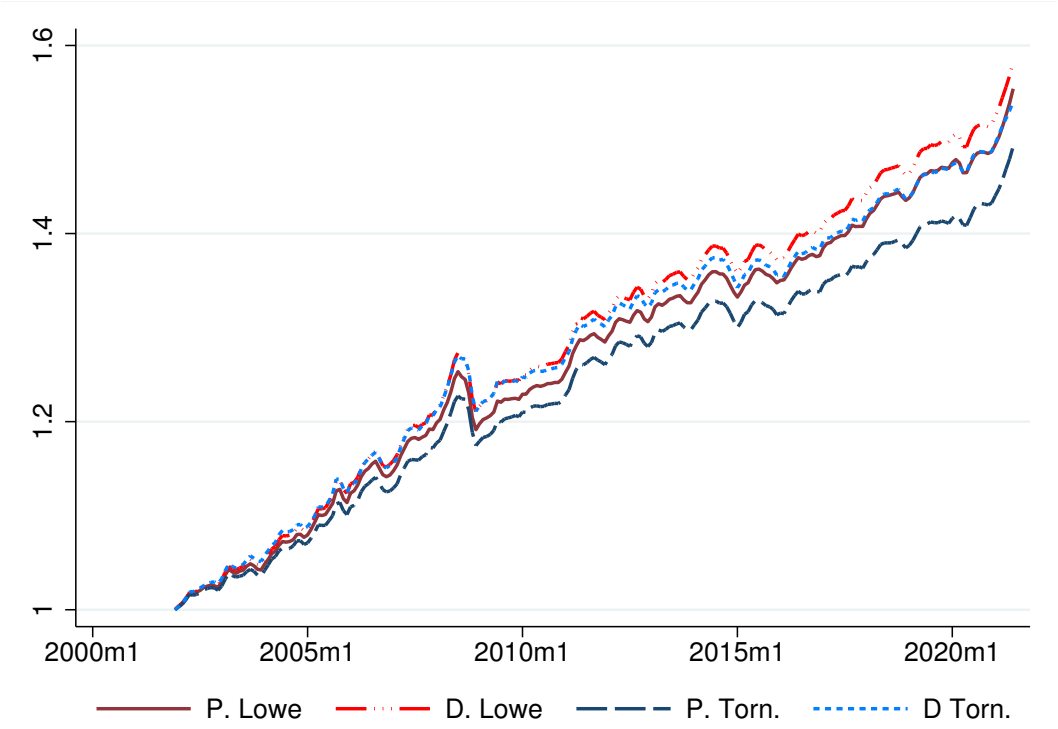
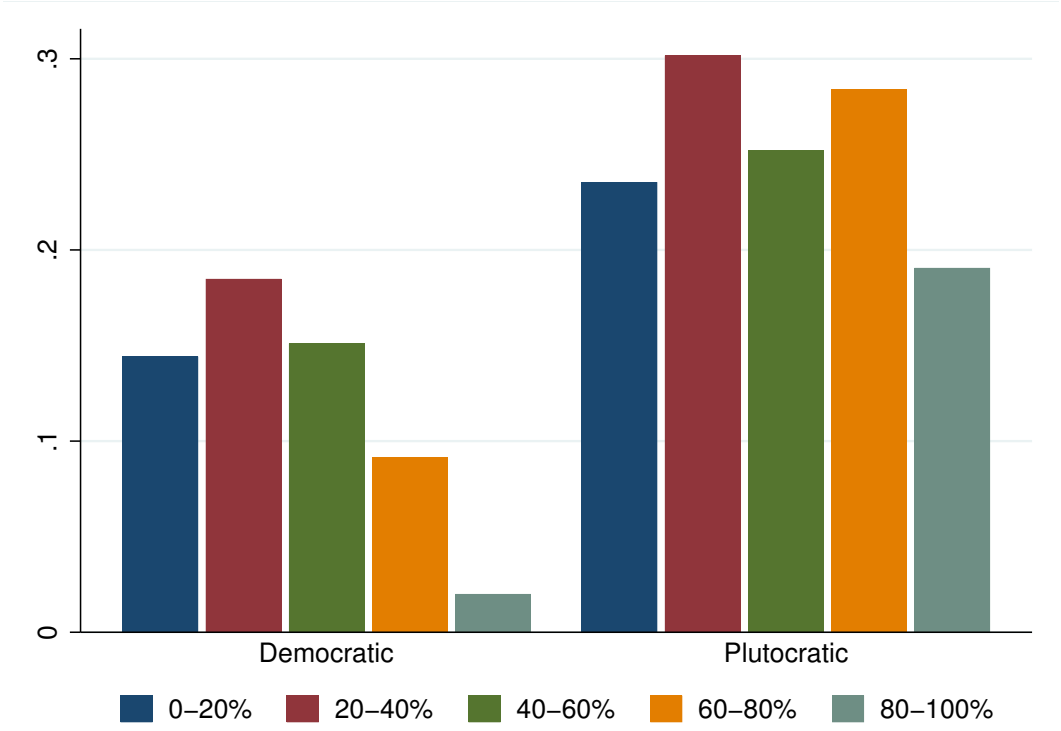
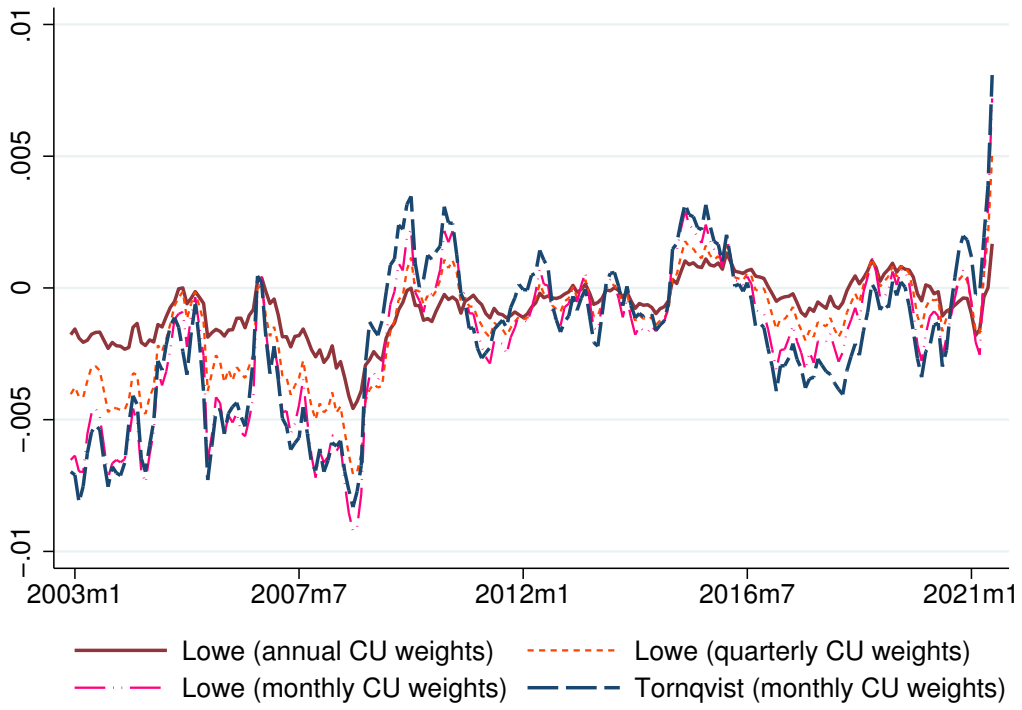


Figure 2: Consumer Unit Indexes: Average Substitution Effects (% point differences)



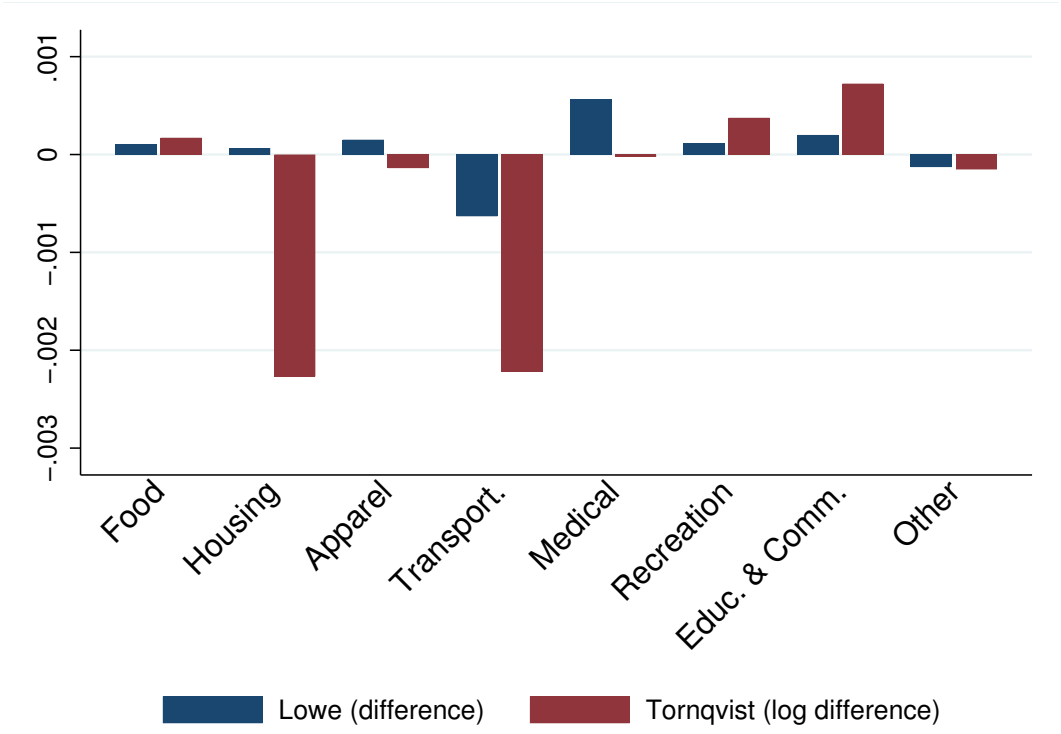
Notes: Bars represent differences between average 12 month percent changes of plutocratic and democratic Lowe and Tornqvist indexes from December 2002 to June 2021. For each index and separately by weight reference period, consumer units are first sorted into quintiles of annual expenditure, using only observations that completed four interviews.

Figure 3: Consumer Unit Indexes: Plutocratic Gaps in 12 month % Changes



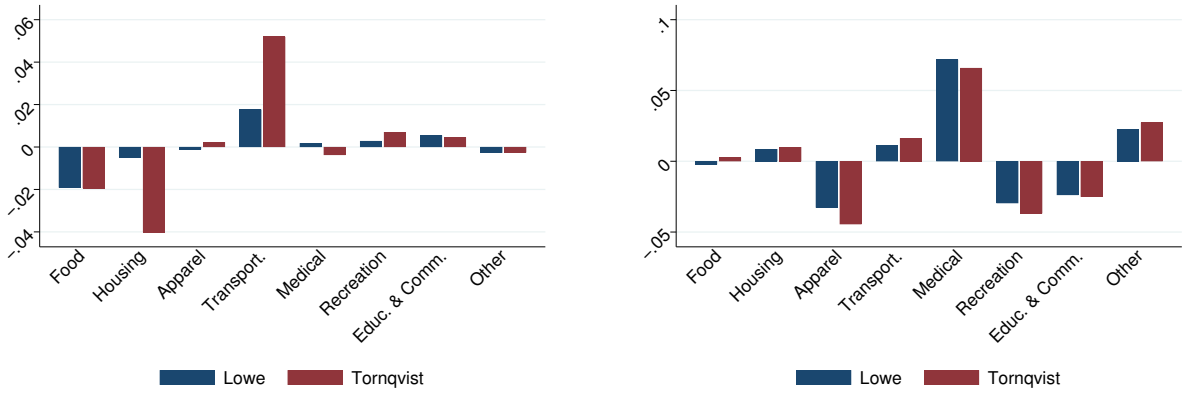
Notes: Plots depict differences between 12 month % changes for comparable plutocratic and democratic indexes based on consumer units as households. Lowe (annual CU weights) and Tornqvist (monthly CU weights) correspond to the indexes presented in Figure 1 and Table 2. Lowe (quarterly CU weights) treats CU-quarter combinations as unique households, while Lowe (monthly CU weights) treats CU-month combinations as unique households.

Figure 4: Consumer Unit Indexes: Contributions to Plutocratic Gaps (2018-19)



Note: For each expenditure category $\mathcal{G}$, the Lowe bars represent $\sum_{i\in\mathcal{G}}(s_{i\{v,b\}} - \bar{s}_{i\{v,b\}})(r_i - \bar{r})$ where v is 2015-2016, $r_i = p_{i,2019m12}/p_{i,2017m12}$ and $\bar{r} = N^{-1}\sum_{i=1}^{N} r_i$. The Tornqvist bars represent $\sum_{t=2018\text{m}1}^{2019\text{m}12}\sum_{i\in\mathcal{G}}(w_{it} - \bar{w}_{it})\left(\ln r_{it} - \overline{\ln r_{it}}\right)$, where $r_{it} = p_{it}/p_{i,t-1}$ and $\overline{\ln r_{it}} = N^{-1}\sum_{i=1}^{N}\ln r_{it}$.

Figure 5: Consumer Unit Indexes: Factors of Plutocratic Gaps (2018-19)

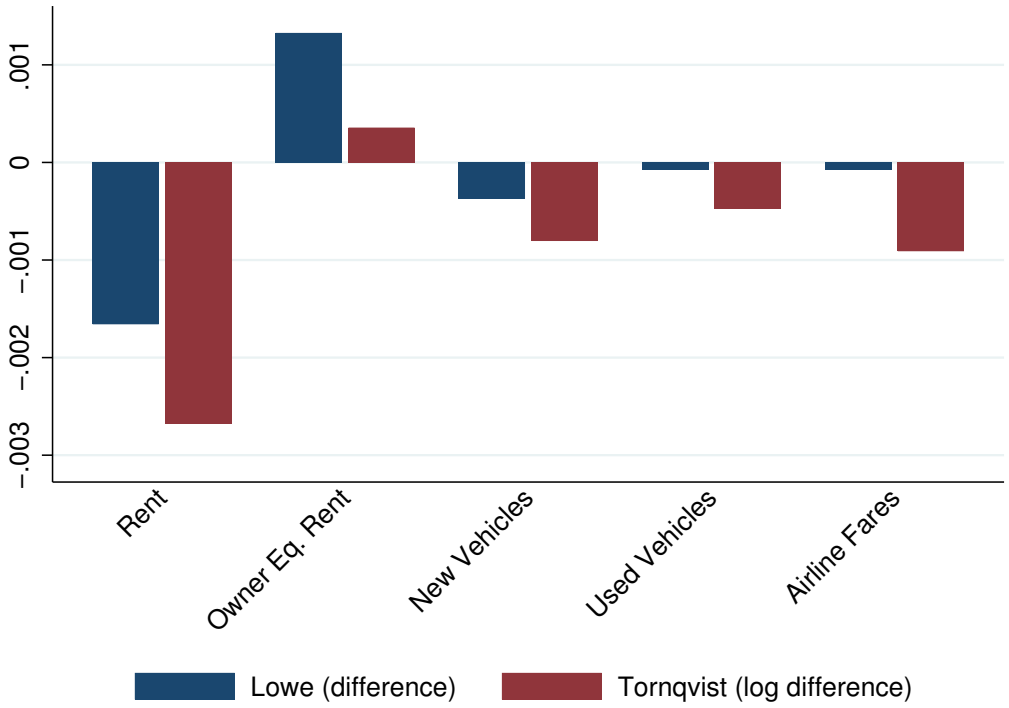


(a) Weight Differences

(b) Inflation Relative to Mean

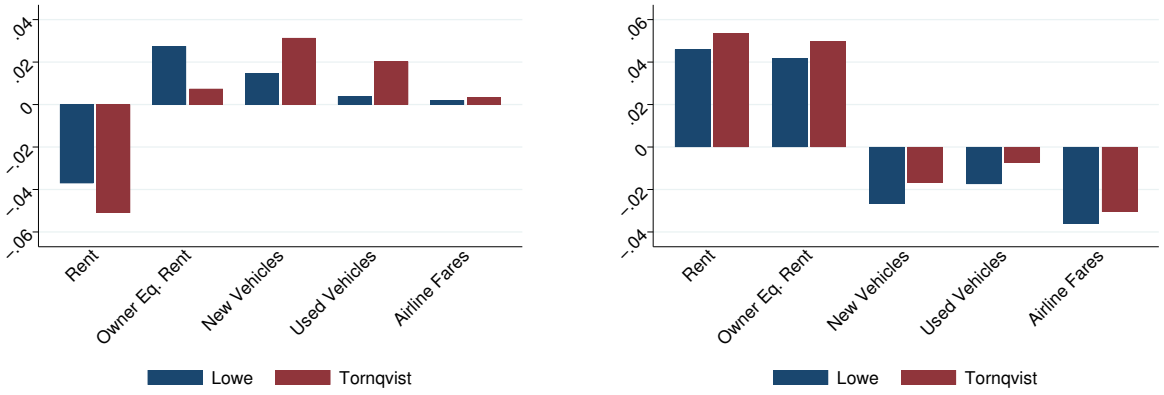
Notes: For each expenditure category $\mathcal{G}$, the Lowe weight differences are given by $\sum_{i\in\mathcal{G}}(s_{i\{v,b\}} - \bar{s}_{i\{v,b\}})$, while the inflation relative to the mean are given by $N_g^{-1}\sum_{i\in\mathcal{G}}(r_i - \bar{r})$ where N_g is the number of elementary cells in category g, b is 2015-2016, v is December 2017, $r_i = p_{i,2019m12}/p_{i,2017m12}$ and $\bar{r} = N^{-1}\sum_{i=1}^{N} r_i$. The Tornqvist weight differences are given by $24^{-1}\sum_{t=2018\text{m}1}^{2019\text{m}12}\sum_{i\in\mathcal{G}}(w_{it} - \bar{w}_{it})$, while the inflation relative to the mean are $\sum_{t=2018\text{m}1}^{2019\text{m}12} N_g^{-1}\sum_{i\in\mathcal{G}}\left(\ln r_{it} - \overline{\ln r_{it}}\right)$ where $r_{it} = p_{it}/p_{i,t-1}$ and $\overline{\ln r_{it}} = N^{-1}\sum_{i=1}^{N}\ln r_{it}$.

Figure 6: Consumer Unit Indexes: Contributions to Plutocratic Gaps by Select Items (2018-19)



Note: For each expenditure category $\mathcal{G}$, the Lowe bars represent $\sum_{i\in\mathcal{G}}(s_{i\{v,b\}} - \bar{s}_{i\{v,b\}})(r_i - \bar{r})$ where v is 2015-2016, $r_i = p_{i,2019m12}/p_{i,2017m12}$ and $\bar{r} = N^{-1}\sum_{i=1}^{N} r_i$. The Tornqvist bars represent $\sum_{t=2018\text{m}1}^{2019\text{m}12}\sum_{i\in\mathcal{G}}(w_{it} - \bar{w}_{it})\left(\ln r_{it} - \overline{\ln r_{it}}\right)$, where $r_{it} = p_{it}/p_{i,t-1}$ and $\overline{\ln r_{it}} = N^{-1}\sum_{i=1}^{N}\ln r_{it}$.

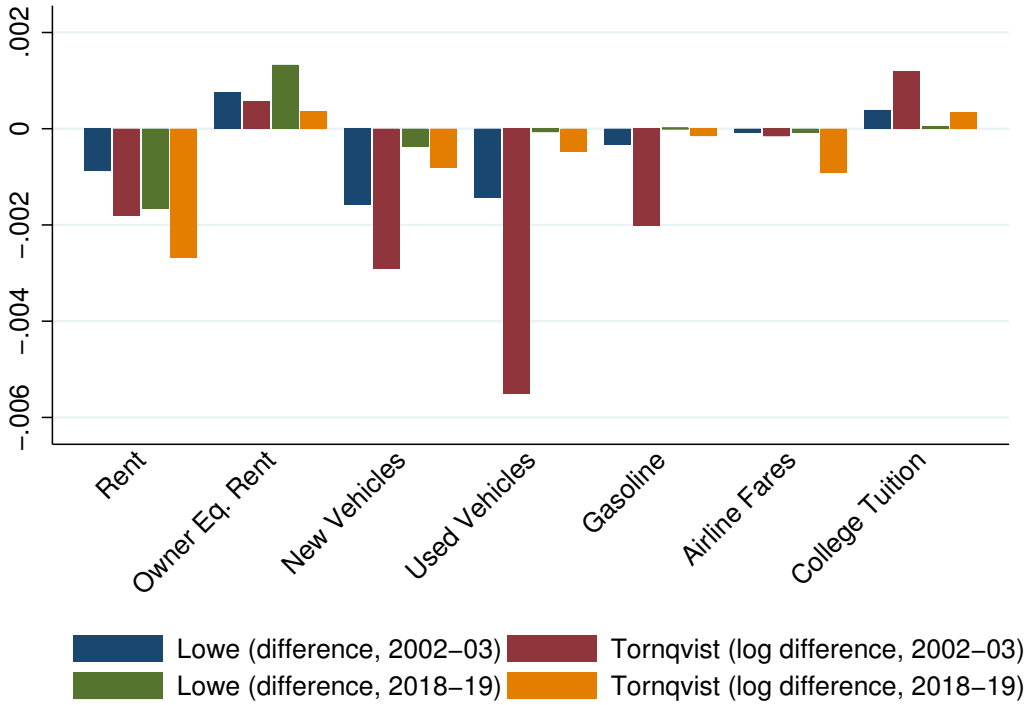
Figure 7: Consumer Unit Indexes: Factors of Plutocratic Gaps by Select Items (2018-19)



(a) Weight Differences (b) Inflation Relative to Mean

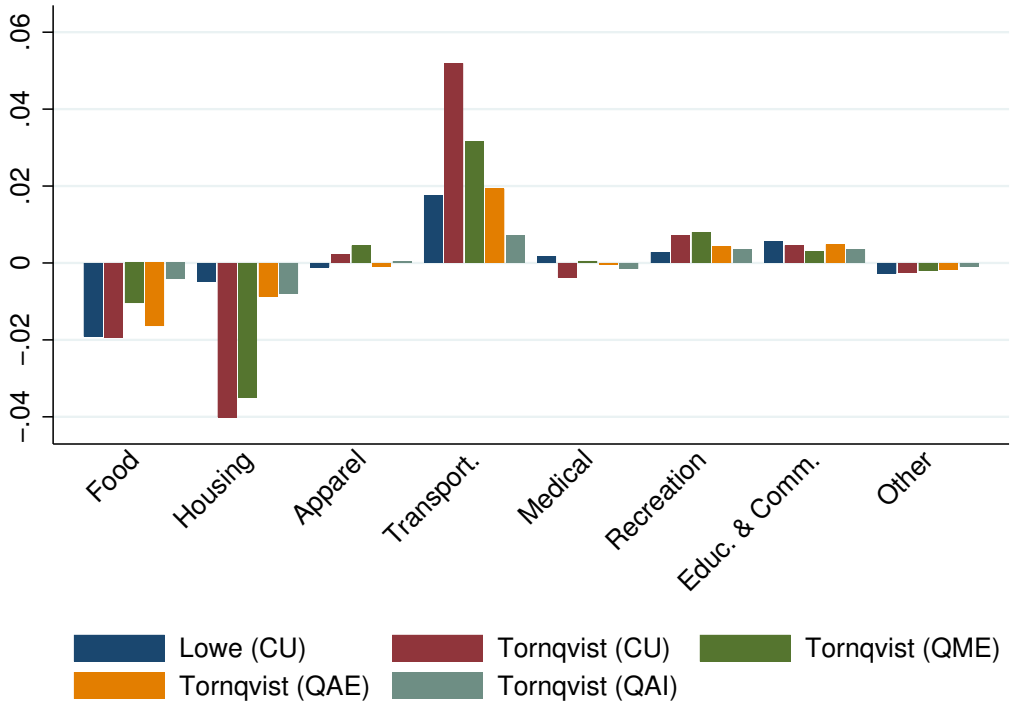
Notes: For each item-stratum $\mathcal{G}$, the Lowe weight differences are given by $\sum_{i\in\mathcal{G}}(s_{i\{v,b\}} - \bar{s}_{i\{v,b\}})$, while the inflation relative to the mean are given by $32^{-1}\sum_{i\in\mathcal{G}}(r_i - \bar{r})$ where 32 is the number of index areas, b is 2015-2016, v is December 2017, $r_i = p_{i,2019m12}/p_{i,2017m12}$ and $\bar{r} = N^{-1}\sum_{i=1}^{N} r_i$. The Tornqvist weight differences are given by $24^{-1}\sum_{t=2018\text{m}1}^{2019\text{m}12}\sum_{i\in\mathcal{G}}(w_{it} - \bar{w}_{it})$, while the inflation relative to the mean are $\sum_{t=2018\text{m}1}^{2019\text{m}12} 24^{-1}\sum_{i\in\mathcal{G}}\left(\ln r_{it} - \overline{\ln r_{it}}\right)$ where $r_{it} = p_{it}/p_{i,t-1}$ and $\overline{\ln r_{it}} = N^{-1}\sum_{i=1}^{N}\ln r_{it}$.

Figure 8: Consumer Unit Indexes: Contributions to Plutocratic Gaps by Select Items



Note: For each expenditure category $\mathcal{G}$, the Lowe bars represent $\sum_{i\in\mathcal{G}}(s_{i\{v,b\}} - \bar{s}_{i\{v,b\}})(r_i - \bar{r})$ where v is 2015-2016, $r_i = p_{i,2019m12}/p_{i,2017m12}$ and $\bar{r} = N^{-1}\sum_{i=1}^{N} r_i$. The Tornqvist bars represent $\sum_{t=2018\text{m}1}^{2019\text{m}12}\sum_{i\in\mathcal{G}}(w_{it} - \bar{w}_{it})\left(\ln r_{it} - \overline{\ln r_{it}}\right)$, where $r_{it} = p_{it}/p_{i,t-1}$ and $\overline{\ln r_{it}} = N^{-1}\sum_{i=1}^{N}\ln r_{it}$.

Figure 9: Plutocratic Gap: Weight Difference Comparisons (2018-19)



Notes: Notes: For each expenditure category $\mathcal{G}$, the Lowe weight differences are given by $\sum_{i\in\mathcal{G}}(s_{i\{v,b\}} - \bar{s}_{i\{v,b\}})$, b is the 2015-16 biennial reference period, and v is December 2017. The Tornqvist weight differences are given by $24^{-1}\sum_{t=2018\text{m}1}^{2019\text{m}12}\sum_{i\in\mathcal{G}}(w_{it} - \bar{w}_{it})$. CU means the consumer unit was the base unit for the democratic index. QME means the quintile of current month expenditure is the base unit, while QAE means the quintile of annual expenditure is the base unit. QAI means the quintile of annual before-tax income is the base unit.

A Online Appendix

A.1 Tornqvist Plutocratic Gap Components

This section applies Ley's (2005) decomposition of the plutocratic gap to Tornqvist indexes in natural logs. The result follows directly from Ley's decomposition of the difference between plutocratic and democratic shares.

In terms of prices and expenditure shares, the gap between the plutocratic and the (geometric) democratic Tornqvist is written:

$$\begin{aligned}\ln P_{PT} - \ln P_{DT} &= \sum_{i=1}^{N} (w_{it} - \bar{w}_{it}) \ln \left(\frac{p_{it}}{p_{i,t-1}} \right) \\ &= \sum_{i=1}^{N} \frac{1}{2} \left[(s_{i,t-1} - \bar{s}_{i,t-1}) + (s_{it} - \bar{s}_{it}) \right] \ln \left(\frac{p_{it}}{p_{i,t-1}} \right) \end{aligned} \tag{15}$$

where $s_{it} = \sum_{h=1}^{H} S_{th} s_{ith}$ and $\bar{s}_{it} = H^{-1} \sum_{h=1}^{H} s_{ith}$ are the plutocratic and democratic expenditure shares, respectively.

From Ley (2005), we have:

$$s_{it} - \bar{s}_{it} = \hat{\zeta}_t \hat{\beta}_{it},\ i = 1, \ldots, N. \tag{16}$$

where $\hat{\zeta}_t$ is the square of the coefficient of variation of household expenditure on all items, and $\hat{\beta}_{it}$ is the OLS estimator of the parameter in the following regression.

$$(s_{ith} - \bar{s}_{it}) = \beta_{it} \left(\frac{e_{th} - \bar{e}_t}{\bar{e}_t} \right), \tag{17}$$

where $e_{th} = \sum_{i=1}^{N} p_{it} q_{ith}$ and $\bar{e}_t = H^{-1} \sum_{h=1}^{H} e_{th}$. Ley discusses how $\hat{\beta}_{it}$ relates to the expenditure elasticity of good i with respect to total expenditure.

Analogous to Ley (2005) Eq. 7, we then have

$$\ln P_{PT} - \ln P_{DT} = \frac{1}{2}\left[\hat{\zeta}_{t-1}\widehat{NCov}\left(\hat{\beta}_{i,t-1}, \ln(p_{it}/p_{i,t-1})\right) + \hat{\zeta}_t\widehat{NCov}\left(\hat{\beta}_{it}, \ln(p_{it}/p_{i,t-1})\right)\right], \quad (18)$$

where $\widehat{Cov}\left(\hat{\beta}_{it}, \ln(p_{it}/p_{i,t-1})\right)$ is the sample covariance across i between $\hat{\beta}_{it}$ and $\ln(p_{it}/p_{i,t-1})$. From this, we can see Ley's requirements for the plutocratic gap: variation in household expenditures (nonzero $\hat{\zeta}_t$ and $\hat{\zeta}_{t-1}$), expenditure pattern variation across expenditure levels (nonzero $\hat{\beta}_{it}$'s and $\hat{\beta}_{i,t-1}$'s), and expenditure pattern variation that is systematically related to item-level price changes, that is nonzero $\widehat{Cov}\left(\hat{\beta}_{i,t-1}, \ln(p_{it}/p_{i,t-1})\right)$ and $\widehat{Cov}\left(\hat{\beta}_{it}, \ln(p_{it}/p_{i,t-1})\right)$.

A.2 Matching Procedure

As mentioned, I use a statistical matching method, based on Hobijn, Mayer, et al. (2009), through which similar donors from the Diary provide the missing data for each Interview consumer unit. Similarity is determined by a model of average monthly expenditure as a function of demographic characteristics. The method implicitly places higher weight on characteristics that more strongly predict expenditures. I used a measure based on average monthly expenditure because I found it to be more highly correlated with consumer unit characteristics than raw monthly expenditures. While borrowing much from Hobijn, Mayer, et al. (2009), my approach differs in one key respect. I use the natural log of average monthly expenditure as the dependent variable for the regression model, while Hobijn, et. al. compute a price-related expenditure change variable using an annual Laspeyres index. I also tried a modified version of this procedure using an average expenditure change variable based on a 12 month modified Paasche index. I found both approaches produced very similar democratic indexes. Some features of the joint distribution between Diary expenditures and demographic characteristics were slightly better replicated in the imputed dataset when using

predicted average monthly expenditures in the distance metric. Further details are later in this section.

Denote the set of months that consumer unit h is in the sample as $S(h)$, and the number of months as T_h. Average monthly expenditure for h is defined $\bar{e}_h = T_h^{-1} \sum_{t \in S(h)} \sum_{i=1}^{N} e_{ith}$. I construct $\bar{e}_h$ for Interview observations using all available CPI-eligible expenditures available in the Interview. These include categories that CPI sources from the Interview and categories available in both surveys, but which the CPI sources from the Diary, such as the more aggregated food and beverage categories. Table A1 describes potential alternative source selection for Diary expenditures in greater detail. Detailed CE categories are known as Universal Classification Codes (UCC). Of the 246 UCC's sourced from the Diary in 2019, 63 were also available in the Interview. Coverage for an additional 137 food and beverage UCC's was also available, but only as a handful of coarse aggregates. In total, 83.5% of Diary-sourced expenditures belong to categories for which some Interview information is available. The actual expenditure estimates from the Interview for these categories are slightly less—totaling about 80% of Diary expenditures in 2019 (See Table A2). Referring to Table A1, $\bar{e}_h$ covers everything but the Diary-only UCC's, which account for only 4% of expenditures in 2019.

Table A1: CPI Expenditure Sourcing and Availability, 2019

Description	# UCC	Share of UCC	Share of Expend.
Sourced from Interview	347	0.5852	0.7673
Sourced from Diary	246	0.4148	0.2327
Only in Diary	46	0.0776	0.0384
Also available in Interview	63	0.1062	0.0502
Aggregates available in Interview	137	0.2310	0.1441
Total	593	1.0000	1.0000

UCC = Universal Classification Code (identifier used by the CE). Diary expenditures used to compute expenditure shares in fourth column.

I estimate a linear regression for each month t and within a subpopulation s (e.g., all urban consumers), where s is the subpopulation of interest for the final index (e.g., all urban

Table A2: CE Survey Expenditures on Items CPI Sources from Diary, 2019

Month	Diary (b. dollars)	Interview (b. dollars)	Ratio
January	127.5	105.1	0.8243
February	129.1	105.2	0.8149
March	138.9	109.5	0.7883
April	156.8	111.4	0.7105
May	156.5	113.7	0.7265
June	133.4	115.8	0.8681
July	139.3	114.8	0.8241
August	136.2	115.1	0.8451
September	146.1	112.8	0.7721
October	148.2	112.2	0.7571
November	132.5	115.7	0.8732
December	163.2	129.2	0.7917
Total	1707.7	1360.5	0.7967

consumers, low-income consumers, etc.), I estimate the following linear regression over the Interview observations.[2]

$$\ln(\bar{e}_h) \equiv \; y_h = \; \boldsymbol{x}_{th}\boldsymbol{\beta}_{st} + u_{ht}. \tag{19}$$

The vector $\boldsymbol{x}_{th}$ includes variables like Census region, urban/rural, age, race, sex, and education of the reference person, family size, and the prior year's annual income. These often do not change between collection quarters, and I implicitly assume that attributes true for the collection quarter are also true for the associated reference months. I used the least absolute shrinkage and selection operator (LASSO) estimate each $\boldsymbol{\beta}$. LASSO is a model selection procedure that introduces a penalty term into the least squares objective function. The basic idea is to reduce over-fitting by only including those variables that have the most predictive power. Instead of estimating 25 parameters for every subpopulation in every month, LASSO essentially picks those that are most predictive of average expenditure. For matching within the all urban population, for example, the average number of parameters chosen was about 8. I found that when compared to weighted least squares, observations paired based on

[2]Alternatively, it might be attractive to estimate average expenditures as a function of characteristics use the Diary sample, as these are what need to be imputed for the Interviews. I find that characteristics explain little variation in weekly Diary expenditures for the sample of Diary consumer units, perhaps due to the short (week-long) recall periods.

LASSO distance measures tended to be more likely to have matching attributes, and the covariance between Diary expenditures and characteristics like income were better preserved in the imputed data.

Let $\hat{\boldsymbol{\beta}}_{st}$ be the slope estimate for subpopulation s in period t. As consumer unit characteristics are available and comparably defined in both surveys,[3] I calculate predicted values, $\hat{y}_{ht} = \boldsymbol{x}_h \hat{\boldsymbol{\beta}}_{st}$, for each Diary and Interview. For a given Interview observation h and Diary observation k, the distance metric is defined as

$$\delta_t(h,\ k) = \left|\hat{y}_{ht} - \hat{y}_{kt}\right|. \tag{20}$$

Within each month and subpopulation, I calculate $\delta_t(h,\ k)$ for all h, k pairs. Then for each h, I randomly select one k from the twenty smallest $\delta_t(h,\ k)$. The random component is intended to ensure a more even distribution of matches across Diary observations. This process is repeated for each month the Interview observation is in the sample. Prior to index calculation, the Diary expenditures are scaled by 13/3 to represent one month. When matching within the urban population, variation in $\boldsymbol{x}_h$ explains, on average, about 58% of the variation in y_h. The average distance between matches (in natural logs) is 0.018, which implies a difference in predicted expenditure between matched observations of about 1.8%. For reference, the average difference in predicted expenditure between two randomly matched observations was about 65%. Note, only the subpopulation s is explicitly conditioned on. Variables in $\boldsymbol{x}_h$ will contribute to the match only to the extent to which they predict average expenditures.

I also tried a closer replication of Hobijn, et. al.'s method using price-related expenditure change as the dependent variable of the model. My version is given by

$$\Delta\bar{e}_{ht} = \bar{e}_h \left[1 - \left(P_{P,h,t,t-12}\right)^{-1}\right] \tag{21}$$

[3]As a check, I tested for equality of means across samples for each characteristic an collection quarter. Using a 5% test, I reject the null hypothesis of equality in about 10% of cases.

where

$$P_{P,h,t,t-12} = \left[\sum_{i=1}^{N} (\bar{e}_{ih}/\bar{e}_h)(p_{it}/p_{i,t-12})^{-1} \right]^{-1} \tag{22}$$

Using $\Delta\bar{e}_{ht}$ as the dependent variable, I found model R-squareds averaged only about 0.14 (matching within the urban population), which is lower than what Hobijn, Mayer, et al. (2009) report. With different dependent variables, the R-squares of the regressions of $\Delta\bar{e}_{ht}$ and $\bar{e}_h$ are not comparable, but lower R-squared does not necessarily imply a lower imputation quality, as predicted expenditures are being used for matching purposes only. If the model has no predictive power, then the procedure resembles more a random match between Diary and Interview observations. Figure A3 compare features of the joint distributions of the matched datasets which make the method based on $\bar{e}_h$ look slightly more attractive. Ultimately, however, Plutocratic Tornqvist indexes using predicted $\Delta\bar{e}_{ht}$ and $\bar{e}_h$ differ from each other and the published C-CPI-U cumulatively by only 0.3% over 20 years, and differences between corresponding democratic indexes are only 0.03%, so the choice does not appear to have much of an impact on the eventual index calculation.

Evaluating match quality is difficult because the monthly expenditures on Diary categories are simply missing for the Interview sample. Nevertheless, I include some subjective diagnostic exercises, following Hobijn, Mayer, et al. (2009) and Webber and Tonkin (2013). The following tables and figures concern matches made within the urban population. To begin with, Table A3 compares the rate at which Interviews are matched with Diaries of the same attribute. If matching were completely random, then the proportion of correct matches would equal (in large samples) the proportion of Diary observations with that attribute. Distance-based matching does not guarantee certain attributes will be correctly matched, but we expect to do at least as well as random. For virtually all cases shown here, distance-based matching delivers a Diary of the same attribute at higher rates than if the match were completely random. For ten of the attributes shown here, including income quintiles, using $\ln(\bar{e}_h)$ delivers higher match rates. In contrast, using $\Delta\bar{e}_{ht}$ delivers higher match rates for nine of these attributes, including for Census Region.

Table A3: Match Rates for Selected Attributes by Match Method

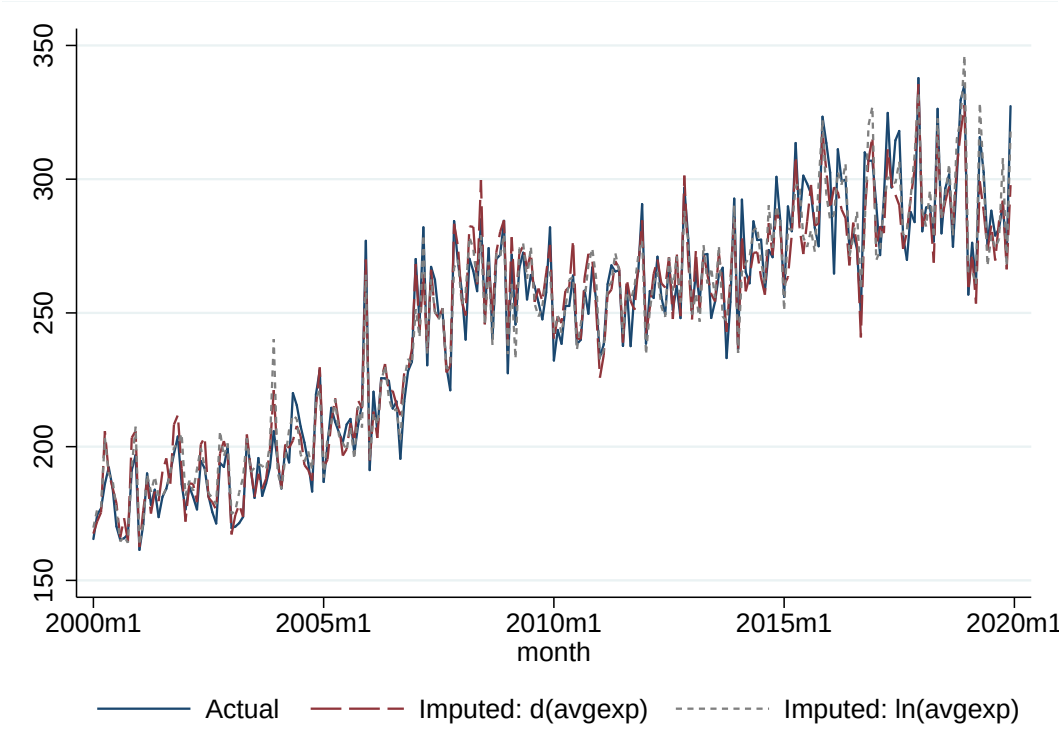
Attribute	Diary Proportion	Match Rates Using $\Delta\bar{e}_{ht}$	Match Rates Using $\ln(\bar{e}_h)$
Age			
< 61	0.7321	0.7528	0.7424
≥ 61	0.2679	0.3246	0.2983
Presence of Children			
Yes	0.4061	0.4638	0.4759
No	0.5939	0.6381	0.6454
Education Level			
Less than H.S.	0.1189	0.1656	0.1960
H.S. & Some Col.	0.5511	0.6150	0.5950
Bachelors+	0.3300	0.5049	0.4938
Census Region			
Northeast	0.1971	0.2200	0.1950
Midwest	0.2356	0.3225	0.2442
South	0.3227	0.3721	0.3273
West	0.2446	0.3944	0.2503
Income Quintile			
0-20%	0.2308	0.5014	0.6200
20-40%	0.1761	0.2959	0.3323
40-60%	0.1830	0.2731	0.2877
60-80%	0.1945	0.2981	0.3257
80-100%	0.2156	0.4575	0.6150
Housing Tenure			
Owner w/ mort.	0.4333	0.5572	0.6340
Owner w/o mort.	0.2133	0.2473	0.2408
Renter	0.3213	0.5302	0.5944

Notes: Statistics calculated following matching within the urban population. The first column of data gives the proportion of Diary observations with a given attribute. The final two columns give the proportion of Interview observations matched with a Diary with the same attribute.

Figures A1 and A2 check that the imputed Diary expenditures (or at least the totals) have distributions that line up with the donor data. Figure A1 plots average weekly expenditures of Diary observations by month against imputed values for each dependent variable used in the distance measure. Figure A2 compares deciles of the actual distribution of average weekly expenditures against deciles of the imputed distributions. Not surprisingly, because the matching methods use actual Diary expenditures, they are able to largely replicate the volatility and distribution of this series as well as its average levels. Figure A3 checks that

features of the joint distributions between average weekly Diary expenditures and certain attributes are replicated in the imputed data. The bars represent average Diary spending for different attributes. Again, none of these attributes are explicitly conditioned on in the matching process, but it is reassuring that certain patterns, e.g., rising Diary expenditures by income quintile, are preserved in the imputed data. While matching based on predicted $\ln(\bar{e}_h)$ does not uniformly outperform matching based on predicted $\Delta\bar{e}_{ht}$ with respect to these joint distributions, it does so for Income. I consider particularly important given the factors contributing to Plutocratic gaps as described in Section 2.

Figure A1: Actual and Imputed Average Weekly Diary Expenditures by Month



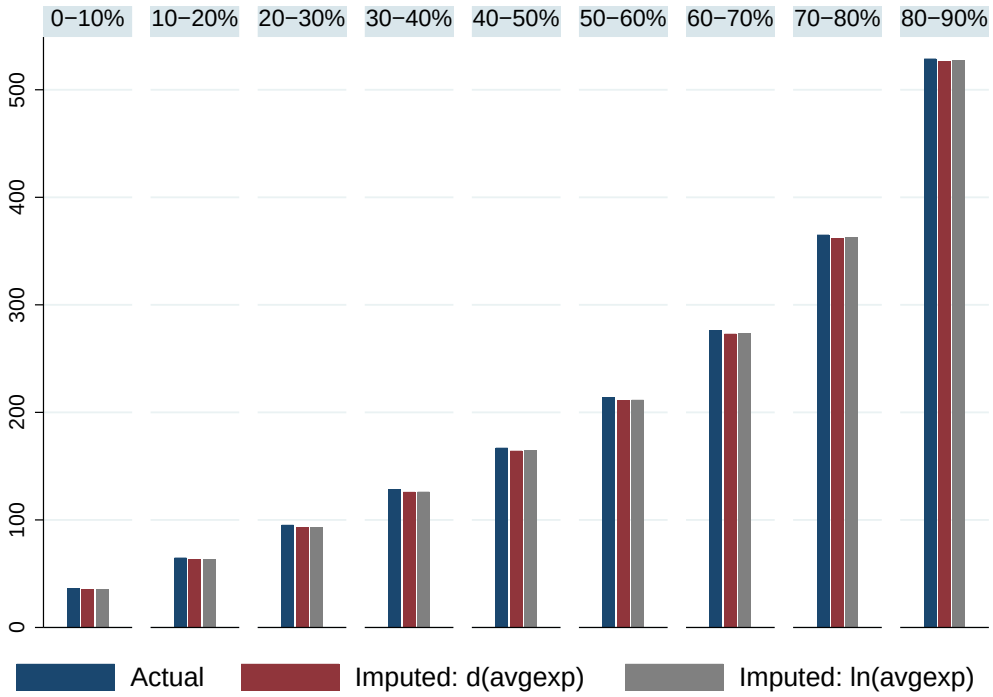
Notes:

Computed following matching within the urban population. d(avgexp) means $\Delta\bar{e}_{ht}$ is the dependent variable used in the distance measure. ln(avgexp) means $\ln(\bar{e}_h)$ is the dependent variable.

Alternatives to Matching

One potential alternative to matching, along the lines of Cage, Klick, and W. Johnson (2018), is to source as much as possible from the Interview survey regardless of classification detail

Figure A2: Deciles of Actual and Imputed Average Weekly Diary Expenditures

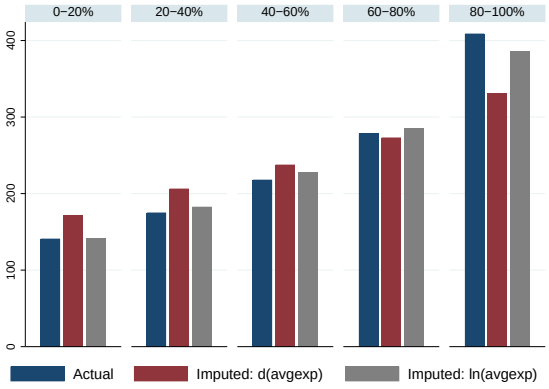


Notes: Computed following matching within the urban population. d(avgexp) means $\Delta\bar{e}_{ht}$ is the dependent variable used in the distance measure. ln(avgexp) means $\ln(\bar{e}_h)$ is the dependent variable.

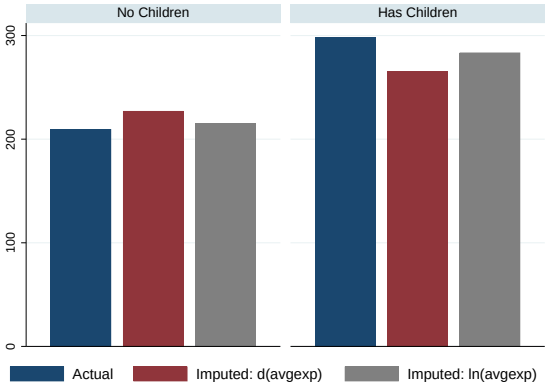
or the CPI program's actual source selection, following Table A1. This results in indexes that reflect roughly 95% of CPI-eligible expenditures. However, they incorporate only a few coarse sub-aggregate indexes for food and beverages rather than the full detail of elementary indexes available. Using this method I find many qualitative aspects of the democratic-plutocratic comparison match my main results (see Figure A11 in this appendix). However, I find it produces plutocratic indexes which differ more significantly from the official CPI and C-CPI than does my replication using the synthetic matched sample. See Online Appendix A.4 for more details.[4]

[4]Another potential alternative, along the lines of Cage, Garner, and Ruiz-Castillo (2002), is to use the Diary to compute average shares of expenditure on detailed food items (e.g., Breakfast Cereal) out of total food spending for some subgroup, and then apply these shares to the total food spending of Interview consumer units within the same subgroup.

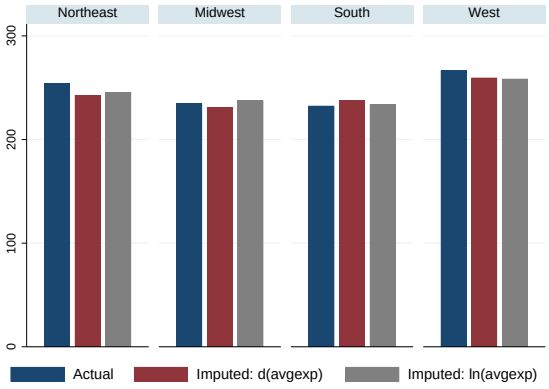
Figure A3: Average Weekly Diary-Category Expenditure By Attribute



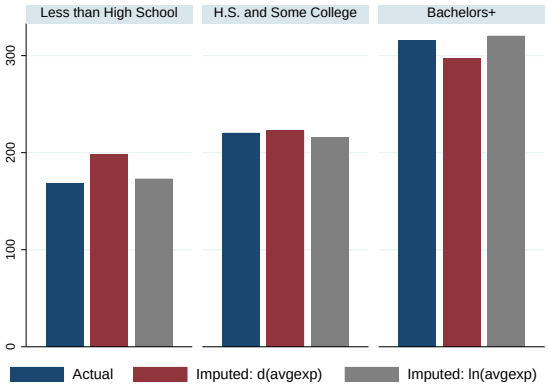
(a) Income Quintiles



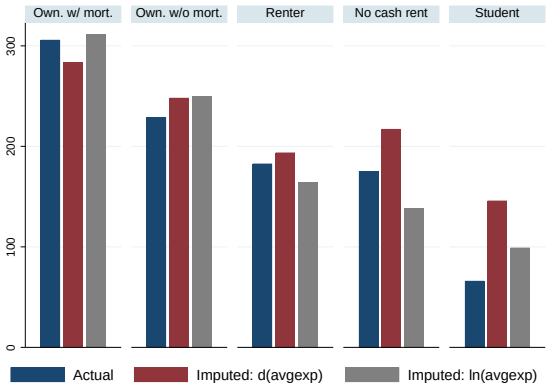
(b) Presence of Children



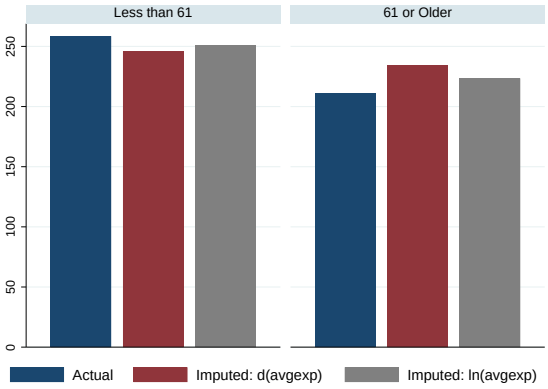
(c) Census Region



(d) Education Level



(e) Housing Tenure



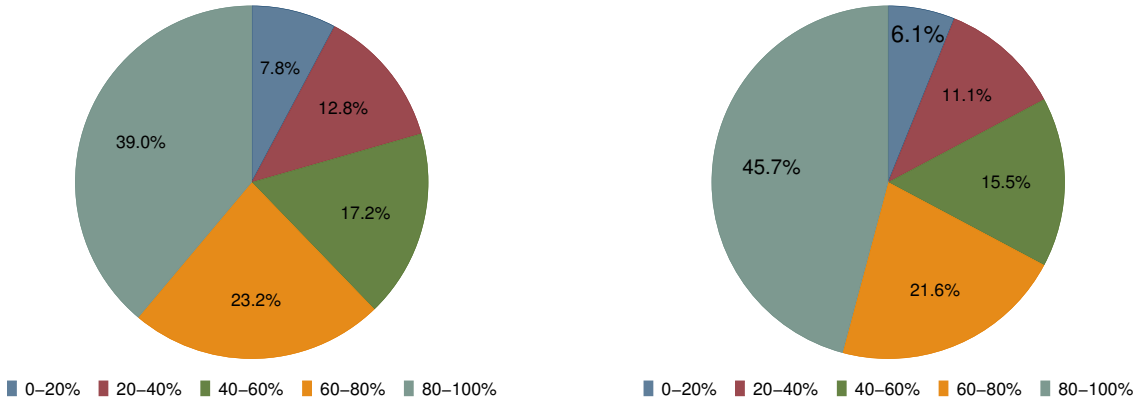
(f) Age

Notes: Computed following matching within the urban population. d(avgexp) means $\Delta\bar{e}_{ht}$ is the dependent variable used in the distance measure. ln(avgexp) means $\ln(\bar{e}_h)$ is the dependent variable.

A.3 Expenditure and Inflation Heterogeneity

As discussed in Section 2.1, a necessary condition for a plutocratic gap to exist is that expenditures are unevenly distributed across households. Figure A4 shows this is true in the CE data. Looking at the expenditures underlying the biennial Lowe index, the top 20% of consumer units accounts for 39% of expenditures over the 2002-2021 sample period, while the bottom 20% accounts for only 7.8%. For the monthly Tornqvist, expenditures are even less equal at the higher frequency, as the top 20% of accounts for 45.7%, while the bottom 20% accounts for only 6.1%.

Figure A4: Consumer Unit Indexes: Reference Period Expenditure Share by Quintile



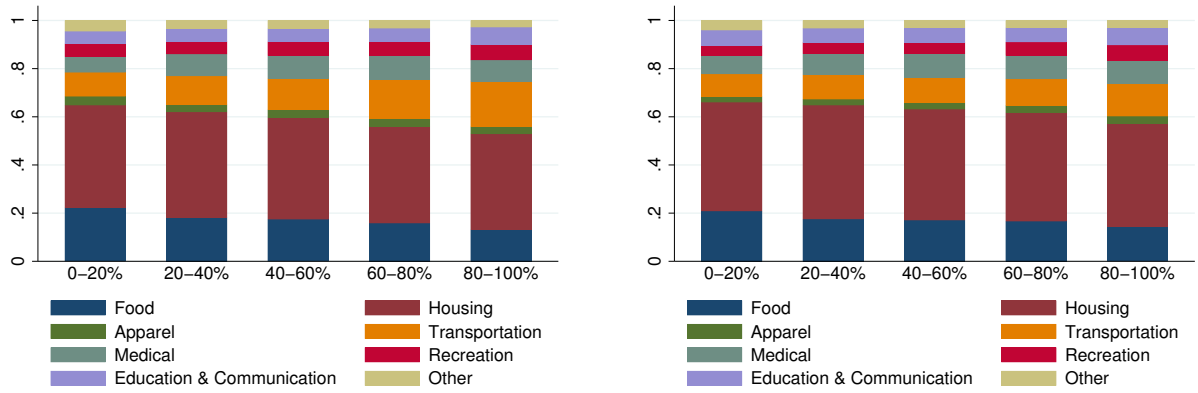
(a) Lowe (b) Tornqvist

Note: Figures depict shares of total reference period expenditures from December 2001 to June 2021 by quintiles of the indexes respective reference period expenditure distributions.

Figure A5 describes how expenditure patterns tend to differ at different points in the expenditure distribution, using the samples of consumer units contributing to the 2018-19 index period (other periods are similar). The stacked bars reflect average expenditure weights within each quintile of the expenditure distribution. Item-level weights are first summed within the eight major categories shown. For the Lowe, the underlying consumer unit expenditures (which occured sometime during the 2015-16 weight reference period) are annual and price-updated to December 2017 (see Section 2). For the Tornqvist, the weights are the average of the consumer unit's current month share and prior month share. Food

weights fall with total expenditure, reflecting a pattern observed at least since Engel in 1857 (Houthakker, 1957). Housing weights also tend to decrease with total expenditure shares, though not monotonically. In contrast, weights for Transportation, Medical Care, and Education and Communication tend to rise with total expenditure, while weights for Apparel are fairly flat. These patterns are broadly consistent with Figure 16 of Cage, Klick, and W. Johnson (2018), which examines annual spending shares by income level and age group for slightly finer categories during the 2013-14 reference period.

Figure A5: Consumer Unit Indexes: Average Expenditure Weights by Quintile (2018-19)



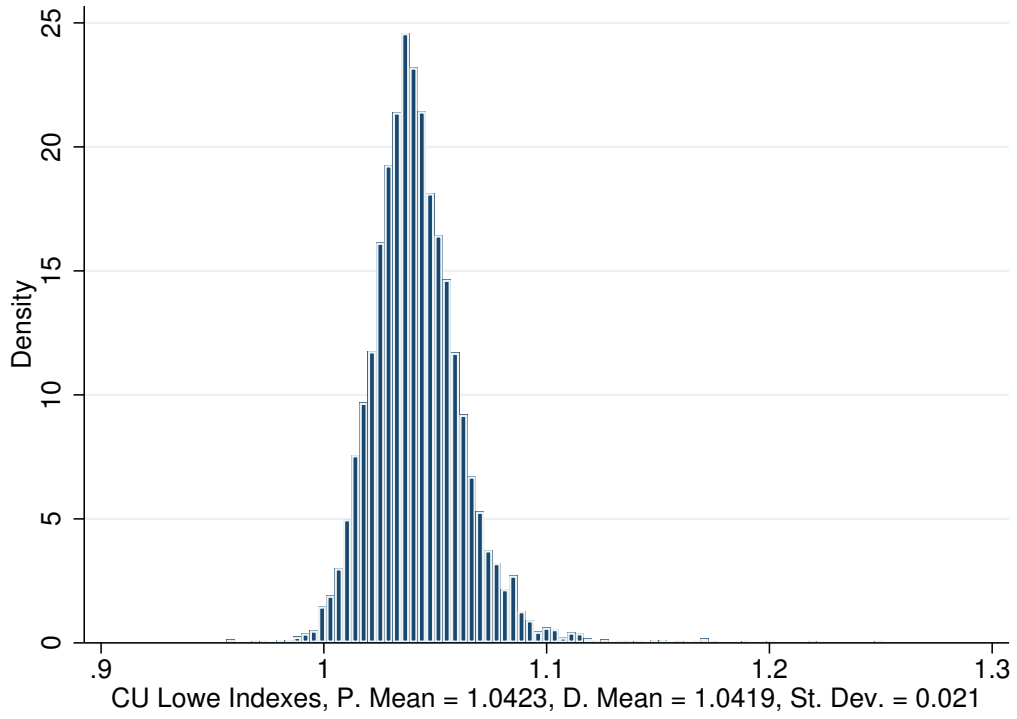
(a) Lowe

(b) Tornqvist

Notes: Bars represent equally-weighted (democratic) average weights of consumer units within the quintiles are of reference period expenditure. For Lowe, the weight is the annual expenditure share (price updated to December 2017) for CU's in the 2015-2016 reference period, corresponding to the 2018-19 index period. For Tornqvist, the monthly weight is itself the average of the consumer unit's current month share and prior month share. Average Tornqvist weights are pooled over January 2018 to December 2019.

Similar to Hobijn, Mayer, et al. (2009) and Kaplan and Schulhofer-Wohl (2017), I find inflation is quite dispersed across consumer units. Figure A6 shows the distribution of Lowe indexes measuring inflation from December 2017 to December 2019 for the 2016-17 sample of consumer units with four interviews. While the plutocratic and democratic means both imply about 4.2% inflation, the standard deviation is about 2 percentage points, with observations ranging from about -5% to 25%. Previous studies such as Kaplan and Schulhofer-Wohl (2017) and Cage, Klick, and W. Johnson (2018) also found considerable dispersion in household-

Figure A6: Consumer Unit Lowe Distribution (Inflation for Dec. 2017 to Dec. 2019)

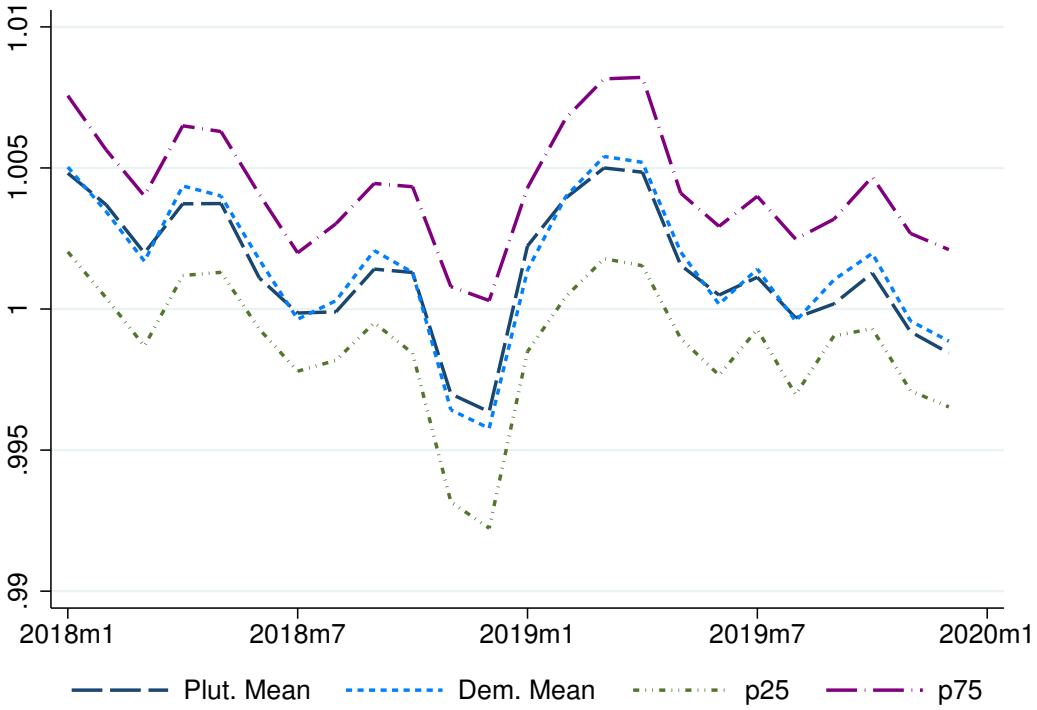


Note: The sample is consumer units with four quarters of expenditure during 2016-17 reference period.

level indexes.[5] Figure A7 summarizes the monthly Tornqvist results over the 2018-19 period, where the consumer level indexes measure one-month inflation. The two means track each other closely, with a tendency of the democratic mean to be slightly higher, though the differences between the plutocratic and democratic mean (-0.015 percentage points on average) are small relative to the interquartile ranges (0.555 percentage points on average). I also find that monthly Tornqvist inflation is not very persistent over time at the consumer unit level. Using a panel first-order autoregression of monthly index relatives in 2019, the autoregressive coefficient is only 0.06 with a standard error of 0.01. On average, consumer unit movements within the distribution of monthly inflation was about 33 percentile points

[5]For comparison, Cage, Klick, and W. Johnson (2018) compute Lowe indexes covering December 2014 to December 2017 for the sample of consumer units with four interviews during 2013-14 and find average inflation of 4.9% with a standard deviation of 1.6 percentage points. Their method is based on coarser Interview aggregates for food spending, which might might explain the lower dispersion. Kaplan and Schulhofer-Wohl (2017) use household scanner data for food and nonfood grocery items over 2004-13 and find an annual interquartile range of 6-9 percentage points.

Figure A7: Monthly Consumer Unit Tornqvist Distributions (previous month = 1.0)



Note: Plots are of means and percentiles of the distributions of household log-Tornqvist indexes by month.

in magnitude each month.

A.4 Additional Analysis for Consumer Unit-Based Indexes

Comparisons with Published Indexes

The democratic and plutocratic indexes calculated for this paper are based on synthetic CE consumer unit observations. As discussed in Section 3, consumer units must be in at least their second month in the sample to calculate a Tornqvist index, and I restrict the sample for the Lowe indexes to those consumer units who completed four interviews within the biennial weight reference period. Section 3 also describes other processing differences between these indexes and the published CPIs, such as the censoring of negative expenditures. Table A4 compares the average twelve-month percent changes of the plutocratic Lowe and Tornqvist indexes against the published CPI-U and C-CPI-U. On average, the matching and processing

differences make a small difference in the plutocratic aggregates. The plutocratic Lowe escalates, on average, 0.05 percentage points less per year than the CPI-U, and the plutocratic Tornqvist escalates, on average 0.02 percentage points per year less than the C-CPI-U. Differences in average twelve month percent changes between the published index and its research counterpart for a specific year are occasionally larger, but these differences are usually less than the estimated plutocratic gaps.

Table A4: Comparison of Published and Research Series Average 12 month Percent Changes

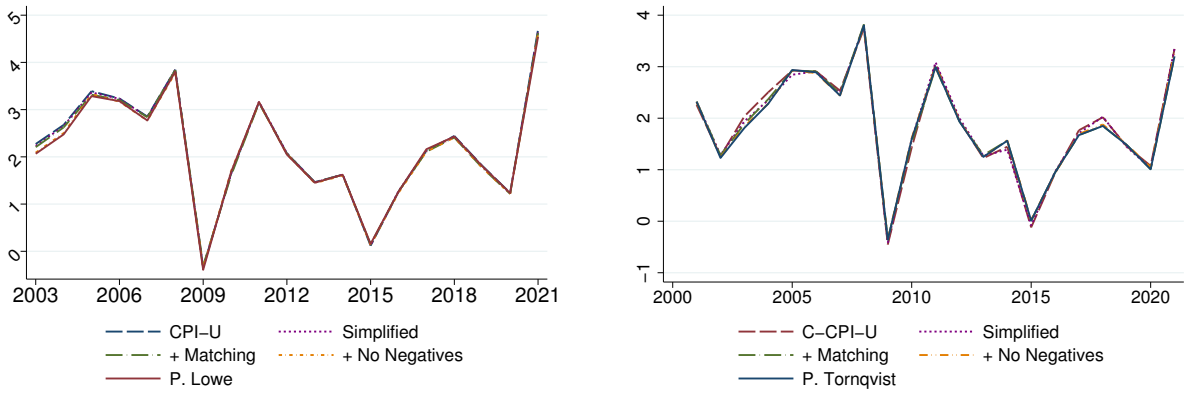
	CPI-U	P. Lowe	C-CPI-U	P. Torn.
2003	2.27	2.07	2.04	1.81
2004	2.68	2.48	2.50	2.28
2005	3.39	3.29	2.93	2.94
2006	3.24	3.18	2.91	2.89
2007	2.85	2.77	2.53	2.44
2008	3.85	3.80	3.74	3.80
2009	−0.34	−0.39	−0.45	−0.36
2010	1.64	1.68	1.43	1.59
2011	3.16	3.16	3.05	2.99
2012	2.07	2.05	1.95	1.94
2013	1.47	1.45	1.23	1.25
2014	1.62	1.62	1.45	1.56
2015	0.12	0.16	−0.12	0.01
2016	1.26	1.26	0.93	0.95
2017	2.13	2.16	1.77	1.67
2018	2.44	2.43	2.02	1.85
2019	1.81	1.80	1.45	1.48
2020	1.24	1.23	1.08	1.01
2021	4.69	4.54	3.35	3.20
Average	2.19	2.14	1.84	1.82

Note: Tornqvist indexes end in June 2021.

Choice of Mean for the Democratic Tornqvist

The democratic Tornqvist is defined in this paper (Eq. 4) as an equally weighted geometric mean of consumer unit-level Tornqvist indexes. Alternatively, one could define the democratic Tornqvist using an arithmetic mean. Using a Taylor expansion, one can show that the proportional difference between the two types of means depends on the dispersion in con-

Figure A8: Consumer Unit Indexes: Comparisons with CPI (Average 12m % Change)



(a) Lowe

(b) Tornqvist

Notes: Simplified means a replication procedure that omits weight-smoothing and combines a few items for the purposes of maintaining a constant item structure over time. Matching means using the simple replication procedure with the matched sample. No Negatives means taking the previous step and restricting expenditures at the CU-item-reference period level to be weakly positive. Finally, the P. Lowe index is estimated using only CUs with four interviews, while the P. Tornqvist uses only CUs with expenditures in adjacent months.

sumer unit-level inflation. Let P_{ADT} be an equally-weighted arithmetic average of consumer unit Tornqvist indexes. From a second-order Taylor expansion of $\ln P_{T,h}$ around the point $P_{T,h} = \bar{P}_T$ where $\bar{P}_T \equiv H^{-1} \sum_{h=1}^{H} P_{T,h} = P_{ADT}$. We have

$$\ln P_{T,h} \approx \ln \bar{P}_T + \frac{1}{\bar{P}_T} \left(P_{T,h} - \bar{P}_T \right) - \frac{1}{\left(\bar{P}_T \right)^2} \left(P_{T,h} - \bar{P}_T \right)^2 \tag{23}$$

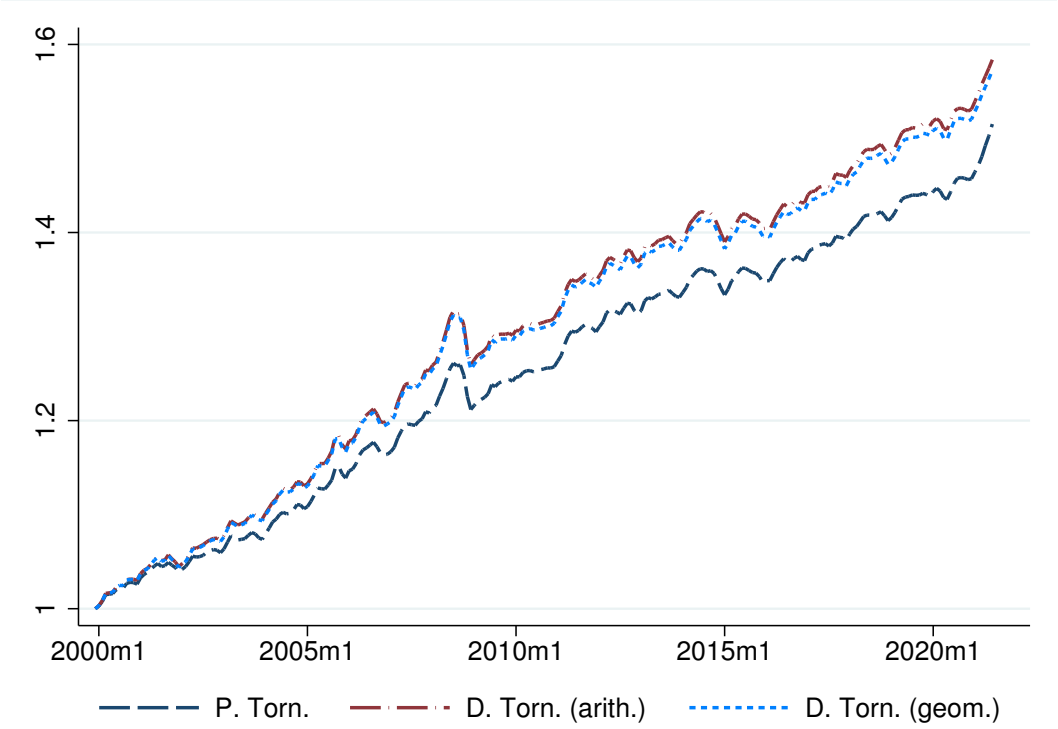
Taking the sample arithmetic mean across h, the first order term drops out. Re-arranging, we have

$$\ln P_{ADT} - \ln P_{DT} \approx \frac{1}{\left(\bar{P}_T \right)^2} H^{-1} \sum_{h=1}^{H} \left(P_{T,h} - \bar{P}_T \right)^2 \tag{24}$$

This shows the difference in logs is approximately equal to the square of the coefficient of variation of the consumer unit Tornqvist indexes, which is weakly positive. As the arithmetic mean is generally greater than the geometric mean, one might be concerned that the choice of mean has a significant impact on the plutocratic gap. Figure A9 implies this is not

the case. With a geometric mean for the democratic Tornqvist, the average gap in twelve month percent changes is -0.22 percentage points per year from December 2000 to June 2021, versus -0.25 percentage points per year when using an arithmetic mean for the democratic Tornqvist.

Figure A9: Democratic Tornqvist: Choice of Mean Across Consumer Units (Dec. 1999 = 1.0)

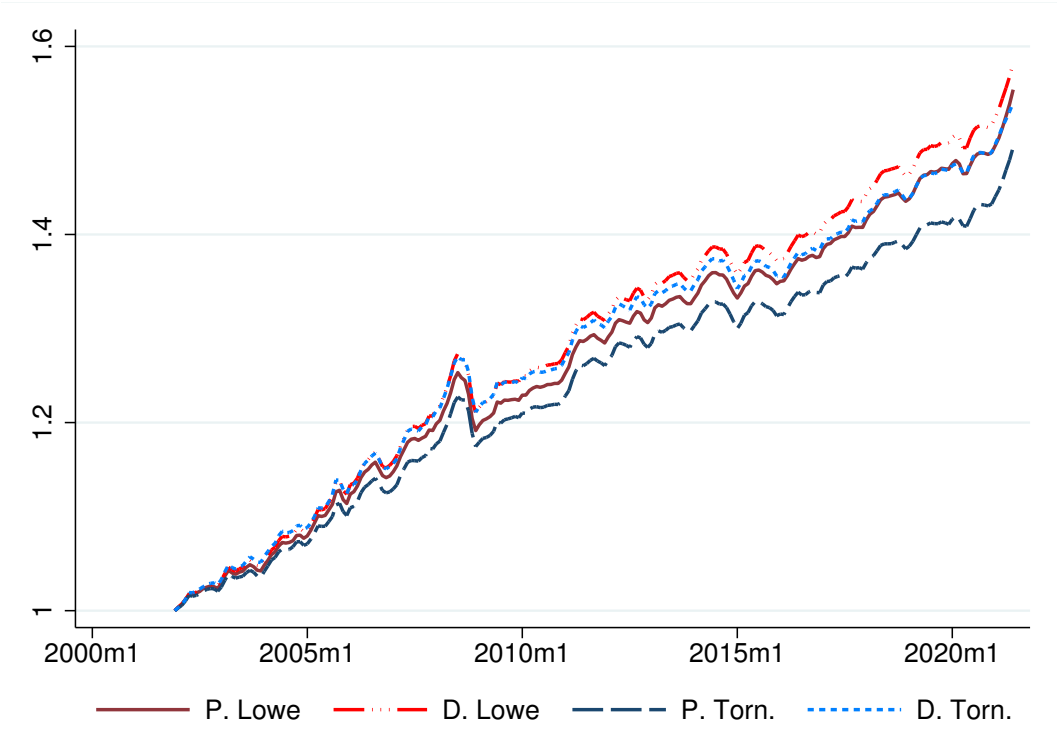


Indexes over CE Interview Expenditures Only

I also calculate the plutocratic and democratic indexes using only Interview-sourced expenditures. This eliminates the need to match Interview and Diary consumer units, but it means that food spending is tracked using only a few broad aggregates. Figure A10 plots the index levels for the plutocratic and democratic indexes as in Figure 1. The average plutocratic gap is slightly larger for the Lowe index, -0.1 percentage points, versus -0.08 in the main results. The average gap is about the same for the Tornqvist index, -0.19 percentage points. While this yields similar gaps as the main results, this is because the gaps are computed

from plutocratic and democratic indexes based on the same data source. The underlying plutocratic indexes turn out to be further from the published CPIs when using only Interview data. Figure A11 shows, for example, that a plutocratic Tornqvist index estimated only from Interview data implies cumulative inflation of about 3.1 percentage points more than the C-CPI-U over the December 1999 to June 2021 period, a compound average difference of about 0.09 percentage points per year.

Figure A10: Consumer Unit Indexes Using Interview Only (Dec. 2001 = 1.0)

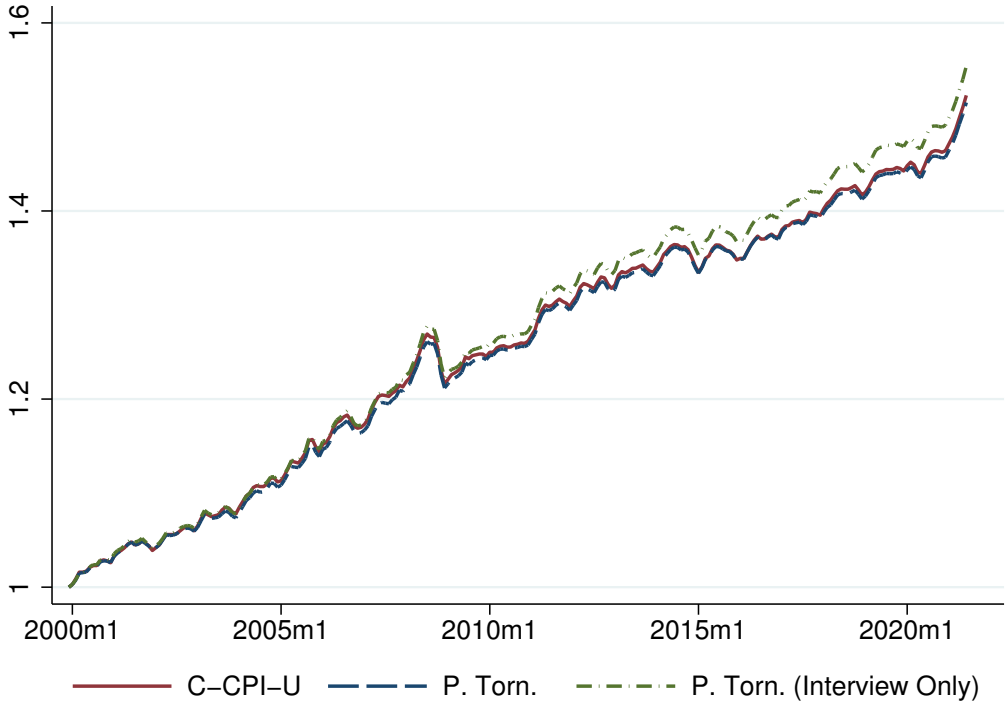


Note: Interview-only indexes use global food questions.

Equivalence Scales

Ley (2005) also considers democratic measures which account for household size in an equivalence scale approach. This is intended to capture potential economies of scale in consumption. The simple model adopted in that paper and here assumes economies of scale depend only on household size. Denote the number of consumer unit members as n_h. We then apply consumer unit-level weights $\omega_h(\theta) \propto n_h^\theta$, where θ is an parameter which governs the

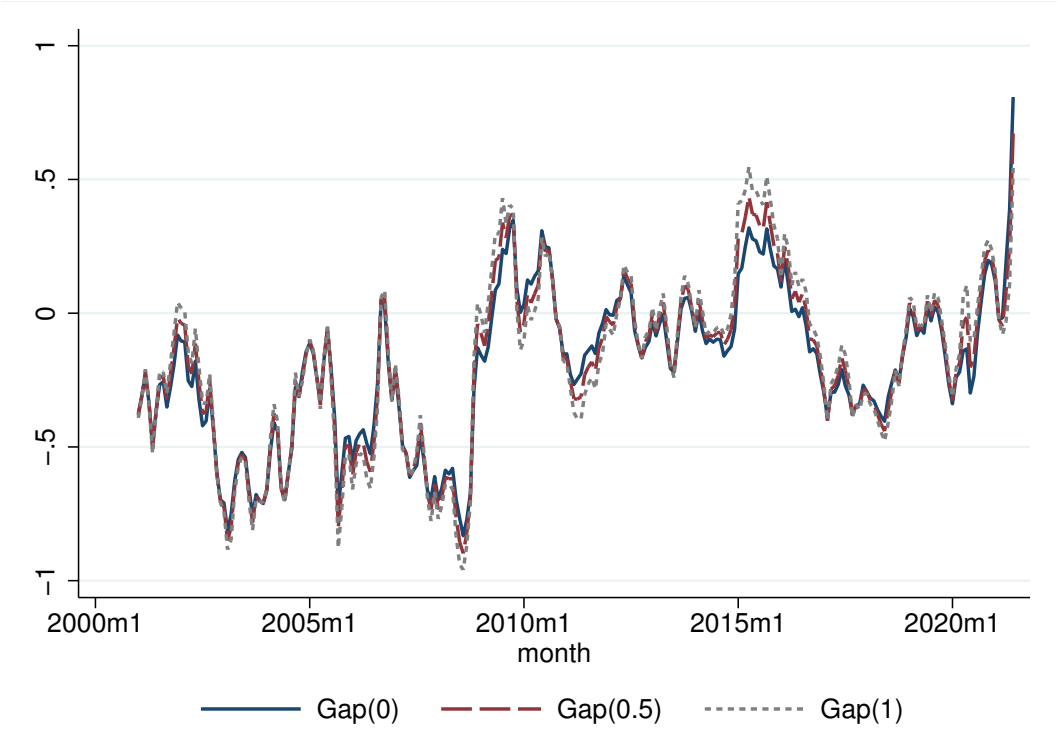
Figure A11: Consumer Unit Indexes: Comparing Survey Source (Dec. 1999 = 1.0)



Note: P. Torn (Interview only) index uses global food questions. P. Torn. uses matched Diary and Interview and same as Fig. 1.

impact of family size on the weight. The democratic index with consumer unit weights $1/H$ corresponds to $\theta = 0$, while $\theta = 1$ uses weights proportional to the full n_h. The latter corresponds to weighting each individual equally rather than each consumer unit. I also consider $\theta = 0.5$, which is a common equivalence scale using the square root of family size. I compute democratic Tornqvist indexes with these levels of θ. Generally, we expect higher θ to push the democratic index closer to the plutocratic in terms of the monthly Tornqvist links, but the twelve month percent changes may not follow the same pattern. Figure A12 shows the twelve month gaps in the Tornqvist indexes for the different θ. Some short-term differences do appear across the values of θ, but the effect of θ is not consistent and tends not to change the plutocratic gap qualitatively. The cumulative inflation implied by the three democratic indexes over December 1999 to June 2021 differ by less than half of one percentage point.

Figure A12: Gaps in Tornqvist 12 month percent changes using equivalence scales



Note: Gaps are differences in 12 month percent changes between plutocratic and democratic Tornqvist. Gap(0) corresponds to the democratic Tornqvist weighting consumer units by $1/H$ as in the main results. Gap(0.5) means consumer unit weights are proportional to the square root of the family size, while Gap(1) means consumer unit weights are proportional to the family size.