Evaluation of Variance Methods for Seasonally Adjusted Series[1] October 2023

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Abstract
There are many papers discussing the estimation of variances for seasonally adjusted data. Serious research extends back to at least the early 1980s, yet there are few instances of official adoption by statistical agencies. Although there are several proposed methods to do so, there are no options in widely used seasonal adjustment software packages. Because of this, users often apply variance measures estimated with not seasonally adjusted data to seasonally adjusted data and assume there are no differences. We apply two methods to estimate variances of seasonally adjusted national Current Population Survey series and compare the results. The first method uses replication variances for seasonally adjusted estimates and the second incorporates linear filter weights based on the seasonal adjustment model.

Key Words: Variances, seasonal adjustment, replication, GVF, sampling errors

1. Introduction

There is much interest in producing confidence intervals for month-to-month changes (MTMC) for national Current Population Survey (CPS) seasonally adjusted (SA) series. BLS produces variance estimates for MTMC of national not seasonally adjusted (NSA) series for many years using generalized variance functions (GVF) (see McIllece, 2019). Most of the proposed methods to create variances for SA series use the X-11 method. Work increased during the 1980s and has continued until now. Regardless, variances for NSA data are often applied to SA data. The focus of this paper is to show that variances based on SA series are doable and useful. This work is an extension of Evans, McIllece, and Miller (2015) where replicate series are seasonally adjusted to create variances. Evans, et al., looked at total employment, total unemployment, and the national unemployment rate.

Brief histories of research on variances for SA series is in Bell and Kramer (1999) and Evans, et al. (2015). Bell and Kramer's method accounts for sampling error (SE) and errors from forecasting extension. Our work here is based primarily on Pfeffermann (1993) with some influences from later work by Pfeffermann, Sverchkov, and others. Pfeffermann's approach can also capture error contributions from the irregular component. Bell (2005) focuses more on contributions of error to the SA error from the regression parameters and other model parameters, forecast extension errors, and parameter estimation errors. Another major difference between Bell and Kramer (1999) and Pfeffermann (1993) is in the definition of the signal and error for a series. Pfeffermann defines the error to be a combined error that makes up the irregular component (irregular + SE). Bell and Kramer differ in this case by assuming the irregular is part of the signal. Pfeffermann and Sverchkov (2014) explain the different approaches in much detail.

The irregular component in most seasonal adjustment procedures can be a combined error component that includes both the irregulars and sampling errors. An option is to estimate the real

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irregular and use its contribution in calculating variances. While most research applies to X-11, our method can easily be applied to model-based procedures such as SEATS or structural time series models. Evans and Sverchkov (2016) use a parametric bootstrap approach for weekly data as the seasonal adjustment model currently used for weekly seasonal adjustment is not suitable for Pfeffermann's approach. Richard Tiller of BLS uses a methodology similar to ours for state CPS employment and unemployment variances, but he utilizes structural time series models with Kalman filtering/smoothing that account for SE (Tiller 2012). Tiller's models produce lower variances for SA series than other methods.

The research in this paper explores the use of replicates to create variances for CPS national SA series. Section 2 describes our data; Section 3 discusses the methodologies used in this paper to create variances; Section 4 covers our results; and Section 5 offers a summary. Figures and tables are in the Appendix and are preceded by references.

2. Data

Fourteen monthly employment (EM) and unemployment (UN) national CPS series were used in this study. The series were selected based primarily on sample sizes to give a range of different noise levels. Each series covers the period from 2003-2022 (same period used for official seasonal adjustments). The EM and UN series are for Total 16+ years, White 16+, Black 16+, Hispanic (or Latino) 16+, Asian 16+, Males 16-19, and Black Females 16-19. Most of the series are not directly seasonally adjusted by BLS for publication, so we seasonally adjusted those series using the SEATS methodology in X-13ARIMA-SEATS (U.S. Census Bureau 2023). Details on how BLS adjusts CPS national series are in Tiller, Evans, and Monsell (2022).

The CPS is a monthly household sample labor force survey conducted by the U.S. Census Bureau for the Bureau of Labor Statistics. The survey covers about 60,000 eligible households that leads to approximately 110,000 individuals each month. The CPS survey has a complex sample design with a 4-8-4 rotation pattern where households are in the sample for four months, out for eight months, and back in for four more months. This pattern is important since it means that approximately 75% of the sample is the same from month to month and 50% from year to year. This rotation pattern thus improves the reliability of estimates for month-to-month and year-to-change.

The Census Bureau provides BLS with 160 replicate weights monthly for each of our CPS national series. The method for creating those weights is consistent back to 2003. We can thus create 160 replicate series for any CPS series to estimate SE covariance matrices and sampling correlations. See Bureau of Labor Statistics and U.S. Census Bureau (2019) for detailed information on CPS methodology.

3. Methodologies

3.1 Linear Approximation to Seasonal Adjustment

Papers such as Pfefferman (1993) discuss linear approximations to estimates of seasonal adjustment components, where the coefficients applied to the not seasonally adjusted series are referred to as observation weights. The first step to constructing observation weights is to create an identity matrix of size N x N. Using the same model and holding the program settings fixed, each column of the identity matrix is seasonally adjusted to get weights for the SA series and the irregular component (X-13 output files *s11* and *s13*). Any outlier effects are removed from the original series and SEATS is run again to get the settings for adjusting the identity matrix columns. Following

Findley and Martin (2006), the SEATS options *imean=0* and *qmax=900* are applied to prevent mean correction of the input series and to prevent SEATS from re-estimating the fixed model coefficients.

A classical seasonal adjustment decomposition at time *t* is:

$$Y_t = T_t + S_t + I_t$$

where Y_t is the observed series without error, T_t is the trend component, S_t the seasonal, and I_t the irregular. For CPS series, we could add another unobserved component,

$$y_t = Y_t + e_t$$

where e_t is the sampling error (SE), independent of Y_t. Tiller (2012) explains this in more detail. Since we cannot explicitly account for SE with the SEATS approach, the irregular component is a combined error containing both the SE and the irregular. Following Pfeffermann (1993), we can either ignore or account for the contribution of the irregular when calculating variances.

The real irregular is computed by using a nonlinear optimization routine to minimize the difference between the irregular and combined variances. The correlations of the irregular can be treated as white noise or low-order moving average processes can be fit to the correlations.

Row *t* of the matrix **W** that follows represents the observation weights for the seasonally adjusted estimate at time *t*. For the observation weight notation, the second index identifies the estimation month, and the first index is the difference between the estimation month and the month that the weight is applied to. For example, the diagonal elements are the weights applied to the NSA estimate for the SA estimate in the same month.

$$\mathbf{W} = \begin{bmatrix} w_{0,1} & w_{1,1} & \cdots & w_{N-1,1} \\ w_{-1,2} & w_{0,2} & \cdots & w_{N-2,2} \\ & \cdots & & \\ w_{-N+1,N} & w_{-N+2,N} & \cdots & w_{0,N} \end{bmatrix}$$

The vector of seasonally adjusted estimates for *t*=1, …, N is approximately $\hat{\mathbf{Y}}_{SA} = \mathbf{W}(\hat{\mathbf{Y}}_{NSA} - \breve{\mathbf{O}})$, where $\hat{\mathbf{Y}}_{NSA}$ is the N-dimensional vector of NSA estimates and $\breve{\mathbf{O}}$ is a vector of outliers. The impact of outliers is important to consider, since large outliers have been seen in our labor force series in recent years, leading to some large differences between $\hat{\mathbf{Y}}_{SA}$ and $\mathbf{W}\hat{\mathbf{Y}}_{NSA}$. Treating the weights and outliers as fixed parameters, the variance of $\mathbf{W}(\hat{\mathbf{Y}}_{NSA} - \breve{\mathbf{O}})$ is equal to the variance of $\mathbf{W}\hat{\mathbf{Y}}_{NSA}$. The results that follow ignore the impact of outliers on variances.

The seasonal component weight matrix is the complement of the weight matrix, **W**.

$$\widetilde{\mathbf{W}} = \mathbf{I} - \mathbf{W} = \begin{bmatrix} \widetilde{w}_{0,1} & \widetilde{w}_{1,1} & \cdots & \widetilde{w}_{N-1,1} \\ \widetilde{w}_{-1,2} & \widetilde{w}_{0,2} & \cdots & \widetilde{w}_{N-2,2} \\ & \cdots & & \\ \widetilde{w}_{-N+1,N} & \widetilde{w}_{-N+2,N} & \cdots & \widetilde{w}_{0,N} \end{bmatrix}$$

The observation weights are used to estimate the variance of any seasonal adjustment component, or the estimate itself, from covariance matrices of the not seasonally adjusted estimates. The results presented in this paper include linear transformations of monthly seasonally adjusted estimates, such as change statistics and averages. For this reason, variance estimators described in prior literature are extended to covariances of the seasonally adjusted estimates.

3.2 Replication Variance Estimators

Variances for both the unadjusted and SA series are calculated using a replicate weighting estimator developed by Bob Fay, involving replication analogues to successive differencing (Fay and Train 1992), and collapsed stratum estimators (Fay 1989). Variances are calculated as:

$$Var(\hat{Y}_0) = \frac{1}{160(1-K)^2} \sum_{r=1}^{160} (\hat{Y}_r - \hat{Y}_0)^2$$

where K=0.5, $\hat{Y}_r$ is the estimate for the r[th] set of replicate weights, and $\hat{Y}_0$ is the full sample estimate using the estimator weights. This approach is followed by both BLS and the Census Bureau for direct estimates of variance.

The replicate series were seasonally adjusted with SEATS holding the model, coefficients, outliers, and other settings fixed. In this paper, the seasonally adjusted estimates were replicated by fixing the observation weights and replicating the not seasonally adjusted estimates.

In April 2014, following a CPS sample design change, the replication variance estimator was redesigned. Correlations between estimates before and after this design change cannot be estimated directly with the replicates. This can lead to discontinuities in variance estimates of seasonally adjusted statistics around those months. To a lesser extent, there may be discontinuities in April 2004, following the previous sample design change.

3.3 Variance Estimation Using Smoothed Covariance Matrices

Replicate variance estimates tend to be noisy. To address this, and to resolve the problems estimating variances around the time of a sample design change, covariance matrices may be smoothed in two ways: modeling the variances of the monthly estimates and modeling the correlation matrices. In monthly production of the BLS series, generalized variance functions (GVFs) are used to smooth out the monthly variances (see McIllece 2019), and correlation matrices are averaged across time. For the numerical results in this paper, the variances using observation weights are not smoothed, but the correlation matrices are averaged, excluding correlations that cannot be reliably estimated due to a change in the variance estimator. Correlations are modeled using an AR (15) model. Tiller (2006) explains the selection of the AR (15) model for CPS SE correlations.

This paper focuses on the use of observation (filter) weights using the current model for seasonal adjustment. BLS uses SEATS for all directly adjusted national CPS series so naturally we use it here. It is also expected that model-based approaches will perform better than X-11 at the tails of the series. Using X-11 can often lead to unreasonable dips in standard errors near the tails of the series. See Scott, Pfeffermann, and Sverchkov (2012) for examples using X-11.

This leads to the following equivalent expressions for the covariance matrix of $\hat{\mathbf{Y}}_{SA}$, Γ, where Λ is the sampling error covariance matrix, and Ω is the irregular error covariance matrix.

$$\Gamma = \Lambda - \widetilde{\mathbf{W}}\Lambda - \Lambda\widetilde{\mathbf{W}}' + \widetilde{\mathbf{W}}(\Lambda + \Omega)\widetilde{\mathbf{W}}'$$
$$\Gamma = \mathbf{W}\Lambda\mathbf{W}' + \widetilde{\mathbf{W}}\Omega\widetilde{\mathbf{W}}'$$

Assuming the irregular error covariances are zero, $\Gamma = \mathbf{W}\Lambda\mathbf{W}'$.

3.4 Change Statistics, Averages, and the Impact of Revisions

Although we are primarily concerned with estimates of the change between the most recent monthly estimate and the previous month's estimate, there are several other statistics that are released each month in the Employment Situation that are of interest. These include changes across different time periods, quarterly and annual averages, as well as changes of averages. The statistics that depend on prior month estimates are based on revisions of those estimates which generally have lower variance than estimates at the end of the series. So, for example, even if a current month's seasonally adjusted estimate has variance similar to that of the not seasonally adjusted estimate, the month-to-month change statistic is likely to have lower variance. The statistics of interest are linear, so can be expressed as $g^T\hat{Y}_{SA}$. A few examples are shown.

Table 1: Examples of Vectors used for Estimation

Statistic	**Estimation vector**
Month-to-month change at the end of the series	$g^T = [0 \quad 0 \quad ... \quad 0 \quad -1 \quad 1]$
Most recent quarterly average	$g^T = \frac{1}{4}[0 \quad 0 \quad ... \quad 0 \quad 1 \quad 1 \quad 1 \quad 1]$
Revised previous quarterly average	$g^T = \frac{1}{4}[0 \quad \dots \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]$

The ratio of the variance of a seasonally adjusted estimate to the not seasonally adjusted estimate is then:

$$vratio = \frac{g^T\mathbf{W}\Lambda\mathbf{W}^{\boldsymbol{T}}g}{g^T\Lambda g}$$

Under the simplifying assumption that the variance of monthly NSA estimates remains constant, the variance cancels out of the numerator and denominator, which leaves:

$$vratio = \frac{g^T\mathbf{W}\hat{\mathbf{R}}\mathbf{W}^{\boldsymbol{T}}g}{g^T\hat{\mathbf{R}}g}$$

Where $\hat{\mathbf{R}}$ is the smoothed correlation matrix. A straightforward approach like this to estimating the variance ratio should help to integrate variance estimates into BLS production systems, which update correlation matrices and GVF estimates for not seasonally adjusted estimates monthly. The frequency with which the variance ratios should be updated needs further study, as does the feasibility of updating factors in a production schedule, which would require recalculating observation weights.

4. Results

Of the fourteen national series chosen for this study, all have a zero or very small irregular components when we explicitly account for SE with a structural time series model. This is confirmed when we account for the irregular when estimating variances for the SA series. Thus, we do not account for the irregular component while calculating variances for these SA series as the combined irregular component in our series are clearly dominated by SE.

Figures 1-14 in the Appendix contain plots of the standard errors for MTMC for the fourteen SA series using Pfeffermann's method with 90% confidence intervals (CIs). The CIs for the replicate weight approach are included for comparisons. As expected, the CIs for the SA series are always lower than for the NSA series. While it varies by series and different time periods, there are many instances where the MTMC value falls in between the CIs for the SA and NSA series. Failing to use standard errors for MTMC changes with SA series can lead data users to assume that changes are not significant.[2]

Table 2 gives the ratios of the variances of the SA series divided by those for the NSA series. The gains for MTMC vary between 15-35 percentage points among the series. Table 2 gives an overall picture of how variances for SA series are different from NSA series.

5. Summary

Major findings are:

- The observation weight approach is doable for many types of seasonally adjusted series and should integrate relatively easily into a production system that currently provides variance estimates only for the NSA estimates.

- Besides the relative simplicity of the approach, it resolves issues that replication estimators have with regard to stability and estimation problems around the phase in of new sample designs.

- Variances for SA CPS series are not equal to those for the NSA series. SA variances are needed, important, and useful.

- More work needs to be done on variances for STS models that account for SE.

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References

Bell, W., and Kramer, M. (1999), "Toward Variances for X-11 Seasonal Adjustments," *Survey Methodology*, 25:1, 13-29.

Bureau of Labor Statistics and U.S. Census Bureau, *Design and Methodology Current Population Survey Design and Methodology Technical Paper 77*, (2019). Washington, DC: Authors. Available online at https://www2.census.gov/programs-surveys/cps/methodology/CPS-Tech-Paper-77.pdf.

Evans, T., McIllece, J., and Miller, S. (2015), "Variance Estimation by Replication for National CPS Seasonally Adjusted Series," in American Statistical Association Proceedings of the Business and

[2] Due to the large effects of the Coronavirus pandemic, some of the plots cut off the peaks and troughs for the MTMC during 2020 to make it easier to see the differences in CIs.

Economic Statistics Section. Available at https://www.bls.gov/osmr/research-papers/2015/st150240.htm.

Fay, R. (1989), "Theory and Application of Replicate Weighting for Variance Calculations," Proceedings of the Section on Survey Research Methods, American Statistical Association, Washington, DC, pp. 212-217.

Fay, R. and Train, G. (1995). "Aspects of Survey and Model-Based Postcensal Estimation of Income and Poverty Characteristics for States and Counties," Joint Statistical Meetings, Proceedings of the Section on Government Statistics, 154-159.

Findley, D., and Martin, D. (2006), "Frequency Domain Analyses of SEATS and X-11/12-ARIMA Seasonal Adjustment Filters for Short and Moderate-Length Time Series," Journal of Official Statistics, 22:1, 1-34.

McIllece, J. (2019), "Expanding Variance Function Coverage in the Current Population Survey," in American Statistical Association Proceedings of the Survey Methods Research Section. Available at https://www.bls.gov/osmr/research-papers/2019/pdf/st190060.pdf.

Pfeffermann, D. (1993), "A General Method for Estimating the Variances of X-11 Seasonally Adjusted Estimators," *Journal of Time Series Analysis*, 15, 85-116.

Pfeffermann, D., and Sverchkov, M. (2014), "Estimation of Mean Squared Error of X-11-ARIMA and Other Estimators of Time Series Components," *Journal of Official Statistics*, 30:4, 809-836.

Scott, S., M., Sverchkov, and Pfeffermann, D. (2012), "Estimating Variance in X-11 Seasonal Adjustment," in *Economic Time Series: Modeling and Seasonality*, edited by W.R. Bell, S.H. Holan, and T.S. McElroy, CRC Press, Boca Raton, FL.

Tiller, R. (2006), "Model-Based Labor Force Estimates for Sub-National Areas with Large Survey Errors," available at https://www.bls.gov/osmr/research-papers/2006/pdf/st060010.pdf.

Tiller, R. (2012), "Frequency Domain Analysis of Seasonal Adjustment Filters Applied to Periodic Labor Force Survey Series," in *Economic Time Series: Modeling and Seasonality*, edited by W.R. Bell, S.H. Holan, and T.S. McElroy, CRC Press, Boca Raton, FL.

Tiller, R., Evans, T., and Monsell. B. "Seasonal Adjustment Methodology for National Labor Force Statistics from the Current Population Survey (CPS)," available at https://www.bls.gov/cps/seasonal-adjustment-methodology.htm.

U.S. Census Bureau (2023), *X-13ARIMA-SEATS Reference Manual* (Version 1.1), Washington, DC: Author. Available at https://www2.census.gov/software/x-13arima-seats/x13as/unix-linux/documentation/docx13as.pdf

Appendix

Table 2: Variance Ratios of SA vs NSA Series

	Total		White		Black		Hispanic		Asian		Black F 16-19		M 16-19	
	EM	UN	EM	UN	EM	UN	EM	UN	EM	UN	EM	UN	EM	UN
Current Month	.973	.939	.967	.939	.907	1.01	.938	1.01	.953	.895	.936	.768	.925	.912
Revisions:														
Previous Month	.839	.829	.840	.828	.914	.733	.894	.722	.931	.877	.928	.762	.901	.896
Two months ago	.798	.777	.799	.777	.918	.696	.872	.690	.914	.865	.917	.761	.886	.883
Three months ago	.780	.765	.780	.765	.918	.684	.860	.678	.903	.855	.908	.757	.874	.874
One year ago	.780	.769	.780	.769	.934	.687	.863	.681	.894	.855	.910	.805	.877	.878
Two years ago	.822	.810	.821	.811	.938	.763	.878	.757	.897	.871	.918	.829	.891	.891
Three years ago	.844	.836	.843	.836	.939	.794	.893	.790	.906	.884	.926	.844	.903	.902
Month-Month Change	.626	.706	.622	.706	.848	.646	.767	.644	.846	.810	.847	.706	.795	.843
Year-Year Change	.996	.995	.994	.995	.992	1.01	.994	1.01	.997	.993	.997	.969	.993	.996
Quarterly Avg	.953	.955	.951	.956	.938	.952	.960	.954	.967	.937	.970	.833	.959	.955
Quarterly Avg Change	.692	.765	.688	.766	.845	.707	.797	.708	.863	.820	.866	.702	.818	.860
Annual Avg	.998	.999	.998	.999	1.00	.998	.999	.999	1.00	1.00	1.00	.999	1.00	1.00
Annual Avg Change	.992	.995	.992	.995	.999	.990	.997	.990	.999	.998	.999	.993	.998	.999

Legend for all Graphs

lci = lower 90% CI, uci = upper 90% CI, rep =replicate, SA = seasonally adjusted, NSA = not seasonally adjusted

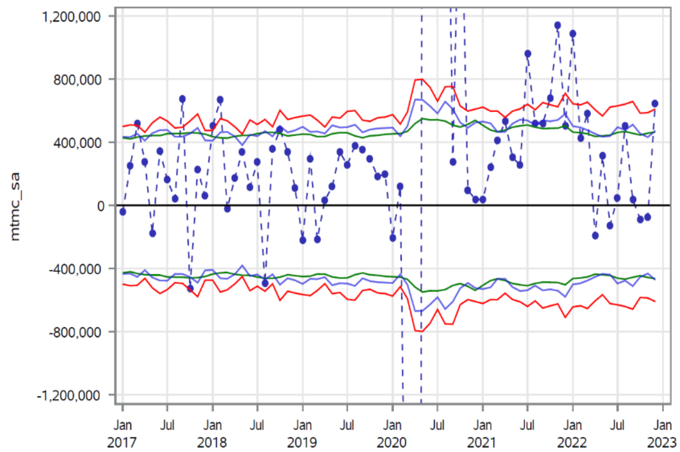

Figure 1: Total Employment

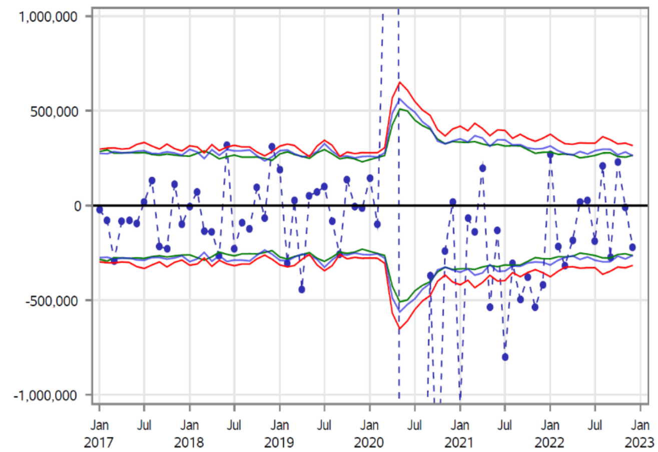

Figure 2: Total Unemployment

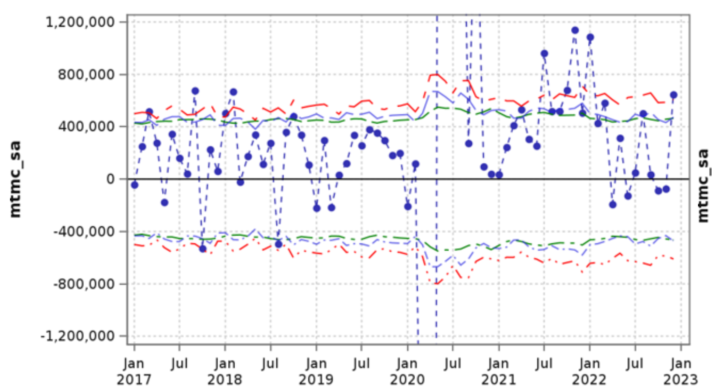

Figure 3: Total Employment, White

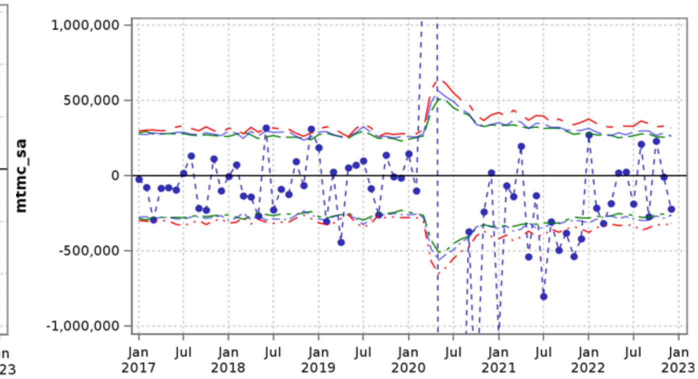

Figure 4: Total Unemployment, White

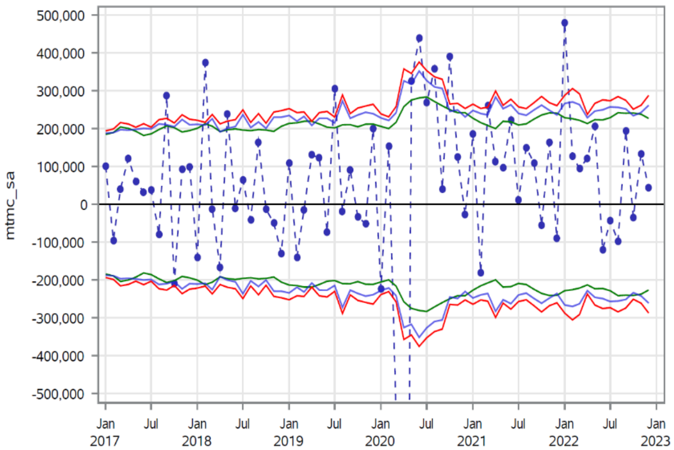

Figure 5: Total Employment, Black

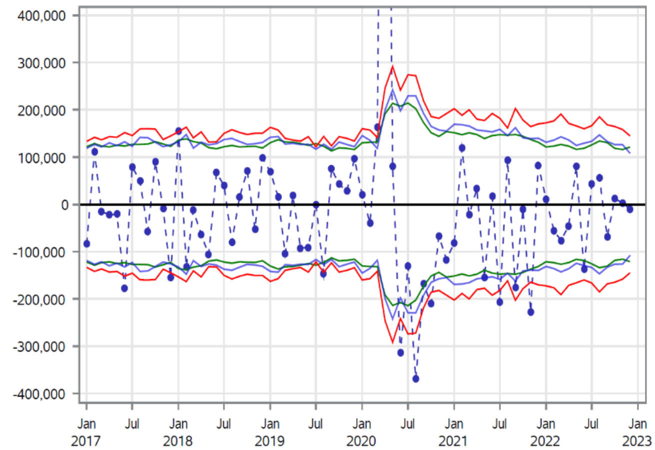

Figure 6: Total Unemployment, Black

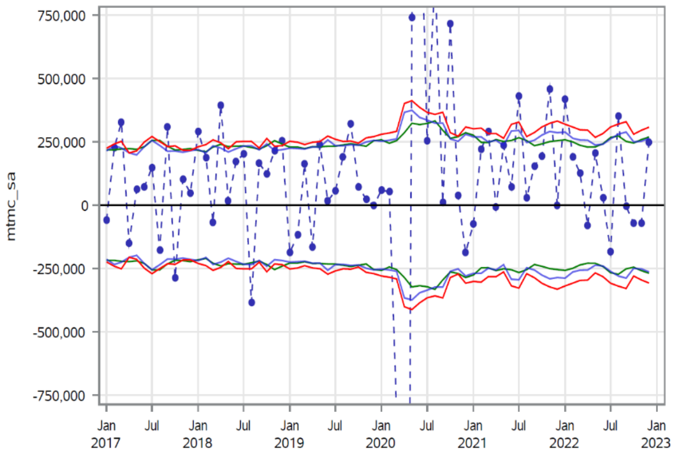

Figure 7: Total Employment, Hispanic

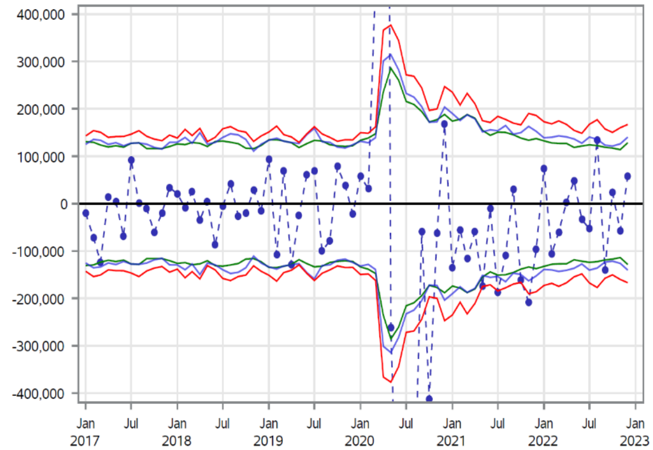

Figure 8: Total Unemployment, Hispanic

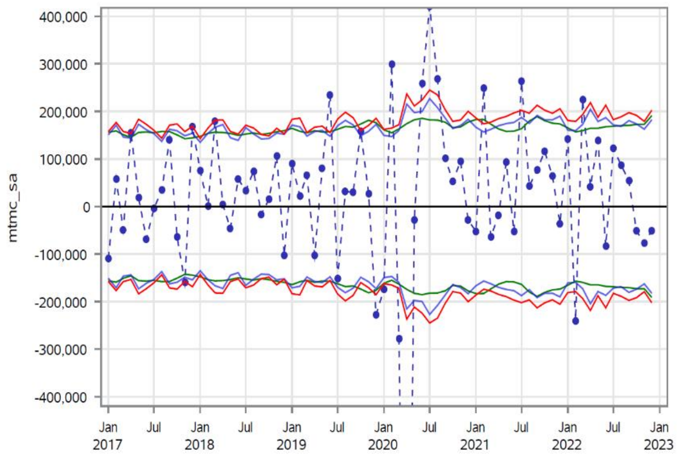

Figure 9: Total Employment, Asian

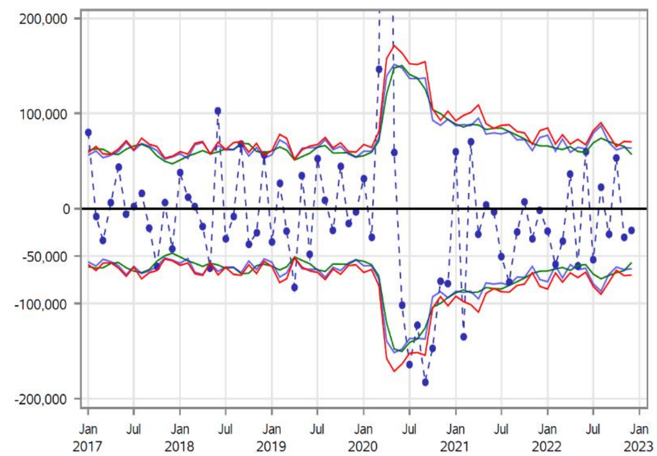

Figure 10: Total Unemployment, Asian

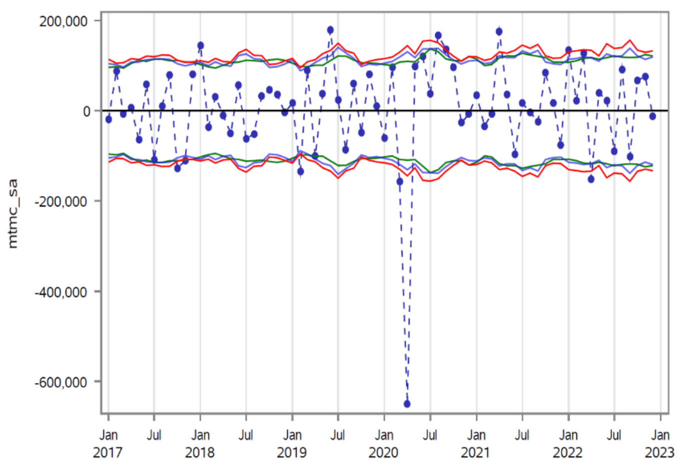

Figure 11: Employment, 16-19

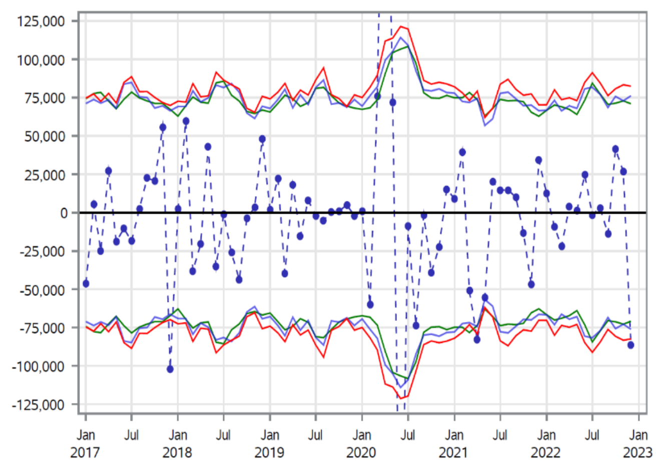

Figure 12: Unemployment, 16-19

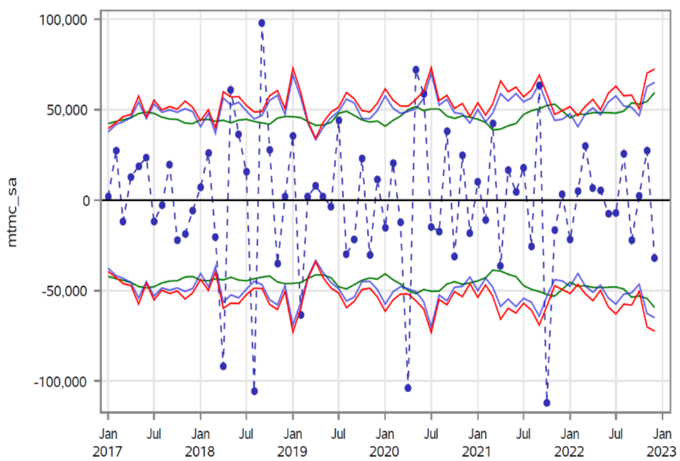

Figure 13: Employment, Black F, 16-19

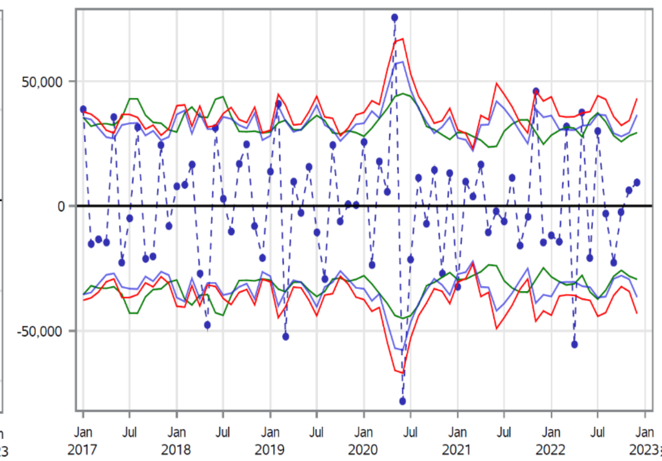

Figure 14: Unemployment, Black, F, 16-19