Standard Errors of Nonstandard Estimates in the Current Population Survey December 2024

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Abstract
Since January 2023, the U.S. Bureau of Labor Statistics (BLS) has produced monthly standard errors for primary Current Population Survey (CPS) labor force estimates using the newly developed GVF (generalized variance function) Production System. In its present form, the GVF Production System computes modeled standard errors for estimates of the following types: levels, or counts; rates, such as the official U.S. unemployment rate; mean and median weeks unemployed; and hourly and weekly earnings percentiles. However, many other CPS labor force series of significant economic interest, such as female-to-male earnings ratios and average hours at work for various demographics, are uncovered by GVF models. In this paper, research models are developed for these "nonstandard" CPS estimates, selected statistical inferences are drawn to evaluate the marginal utility of modeling the variances relative to direct replication, and the potential for implementation into the GVF Production System is discussed.

Key Words: Current Population Survey, CPS, Generalized Variance Function, Replication, Variance Estimation

1. Introduction

For complex multi-stage surveys such as the Current Population Survey, formulaic computation of variance estimates is not always possible let alone an effective and efficient option. Thus, the Current Population Survey has historically used replication methods to improve the efficiency and stability of variance estimates. From 1947 to 2015, the CPS primarily computed these estimates using generalized variance models relating the relative variance to the point estimate for cross-sectional clustered sets of labor force series (Census 2006), presumed to have similar design effects[1]; see Valliant (1987) for a theoretical discussion of this model. However, CPS relative variance estimates tend to be quite noisy and, empirically, dissimilarity of design effects within clusters was apparent, in some cases leading to negative variance estimates, which served as motivation to begin shifting these estimates to a single-series form of the GVF model in August of 2015 (McIllece 2016). The single-series model utilizes the longitudinal history of the series as the clustering method, better ensuring that design effects within clusters are not substantially variable

Currently, the Current Population Survey publishes variances estimates for of the following types: levels, or counts; rates, such as the official U.S. unemployment rate; mean and median weeks unemployed; and hourly and weekly earnings percentiles (McIllece 2018, 2019). When models fit well and enough response data is available, generalized variance functions – mathematical models that describes the relationship between population estimates and their estimators' variances (Wolter 2007) – improve efficiency and usability while also smoothing out noise. Additionally, because much more historical data is used than in past models, the GVFs deployed by the CPS produce time-

[1] Typically, up to a few years of data were combined for modeling to mitigate the effect of changing population size on the variance estimates. Thus, the *a* and *b* coefficients produced by these GVFs degraded quickly and did not generalize well to other time periods.

robust estimates that can retain seasonality features. These models also allow CPS to produce and publicly release more variance estimates in the monthly releases.

To develop a GVF model, first estimates for the population values of interest and their estimated variance must be directly calculated. Then, a model based on the variance or relative variance estimator is developed. The model chosen is often a pragmatic balance between theoretical support and convenience and utility of form, but the development for complex estimates is nontrivial. In section 2, current research into GVF modeling for ratios of percentiles is presented, and in section 3, ongoing research into other estimate types is briefly discussed.

2. GVF Model for Ratio of Percentiles

Expanding upon the existing generalized variance models in production for the Current Population Survey, this paper focuses on nonstandard estimates: namely ratios of percentiles. In application, the model can provide variance estimates for the ratio of women to men median weekly earnings of full-time wage and salary (FTWS) workers as well as the ratio of foreign to native born median weekly earnings of FTWS workers[2]. BLS does not currently provide public variance estimates for either of these series.

Recalling the prior research of McIllece (2019), the framework of the generalized variance function for a percentile was based on equating its replicate variance and its asymptotic variance (under Central Limit Theorem conditions), modified by a design effect:

$$V_r(x_q) \cong \frac{q(1-q)d_q}{y \cdot f(x_q)^2} \tag{1}$$

Derived from successive difference replication (REF – Fay/Train) and a replication form of the collapsed stratum estimator, the direct replicate variance $V_r(x_q)$ is then equated to the modified asymptotic variance given in (1), where q is the percentile value[3], d_q is the design effect, y is the universal base (total FTWS workers), and $f(x_q)^2$ is the squared density function.

Let $p = \left(x_{q,1}/x_{q,2}\right)$ be the ratio of the numerator earnings $x_{q,1}$ and the denominator earnings $x_{q,2}$.

To extend (1) to a ratio of percentiles, a Taylor series expansion was applied to p:

$$V_r\left(\frac{x_{q,1}}{x_{q,2}}\right) \cong \left(\frac{x_{q,1}}{x_{q,2}}\right)^2 \left[\frac{1}{x_{q,1}^2} V_r\left(x_{q,1}\right) + \frac{1}{x_{q,2}^2} V_r\left(x_{q,2}\right)\right] - 2\left(\frac{p}{y^2}\right) Cov\left(x_{q,1}, x_{q,2}\right) \tag{2}$$

Assuming that there is no correlation between the numerator and denominator percentiles, the expansion simplifies to:

$$V_r(p) \cong \frac{1}{x_{q,2}^2} V_r\left(x_{q,1}\right) + \frac{x_{q,1}^2}{x_{q,2}^4} V_r\left(x_{q,2}\right) \tag{3}$$

Further assuming the design effects $d_{q,1} \cong d_{q,2} = d_q$ and density functions $f\left(x_{q,1}\right) \cong f\left(x_{q,2}\right) = f(x_q)$ are approximately equal:

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[2] The model generalizes to other ratios of earnings percentiles. These two series are highlighted because of their appreciable economic interest.

[3] The CPS produces estimates for 10th, 25th, 50th, 75th, and 90th percentile weekly earnings for various series, which correspond to q values of 0.10, 0.25, 0.50, 0.75, and 0.90, respectively.

$$V_r(p) \cong \left(\frac{1}{x_{q,1}^2}\right)\frac{q(1-q)d_q}{y\cdot f(x_q)^2} + \left(\frac{x_{q,1}^2}{x_{q,2}^4}\right)\frac{q(1-q)d_q}{y\cdot f(x_q)^2} \tag{4}$$

$$= \frac{\left(x_{q,1}^2 + x_{q,2}^2\right)}{x_{q,2}^4}\cdot\frac{q(1-q)d_q}{y\cdot f(x_q)^2}$$

The difficult components of (4) to estimate are the design effect and the density function, so these terms are estimated by replication:

$$V_r(p) \cong \frac{\left(x_{q,1}^2 + x_{q,2}^2\right)}{x_{q,2}^4}\cdot\frac{q(1-q)d_q}{y\cdot f(x_q)^2} \rightarrow \frac{V_r(p)\cdot y}{q(1-q)}\cdot\frac{x_{q,2}^4}{\left(x_{q,1}^2 + x_{q,2}^2\right)} \cong \frac{d_q}{f(x_q)^2} \tag{5}$$

To create a model for the *d*-adjusted inverse density function, (5) can be formulated as a GVF model:

$$\frac{V_r(p)\cdot y}{q(1-q)}\cdot\frac{x_{q,2}^4}{\left(x_{q,1}^2 + x_{q,2}^2\right)} \cong \frac{d}{f(x)^2} = \alpha_0 x_{q,2}^2 + \beta_0 x_{q,1}^2 \tag{6}$$

To simplify the above, alpha and beta are defined to absorb the $q(1-q)$ term:

$$\alpha = q(1-q)\alpha_0$$
$$\beta = q(1-q)\beta_0$$

Thus, (6) can be rewritten as:

$$\hat{V} = \frac{\left(x_{q,1}^2 + x_{q,2}^2\right)}{x_{q,2}^4}\cdot\frac{q(1-q)\cdot\left(\alpha_0 x_{q,2}^2 + \beta_0 x_{q,1}^2\right)}{y} \tag{7}$$
$$= \frac{\left(x_{q,1}^2 + x_{q,2}^2\right)}{x_{q,2}^4}\cdot\frac{\left(\alpha x_{q,2}^2 + \beta x_{q,1}^2\right)}{y}$$

This can further simplify by recalling that $p = \left(x_{q,1}/x_{q,2}\right)$:

$$\hat{V} = \frac{\left(x_{q,1}^2 + x_{q,2}^2\right)}{x_{q,2}^4}\cdot\frac{\left(\alpha x_{q,2}^2 + \beta x_{q,1}^2\right)}{y} = \frac{\left(x_{q,1}^2 + x_{q,2}^2\right)}{x_{q,2}^2}\cdot\frac{\left(\alpha x_{q,2}^2 + \beta x_{q,1}^2\right)}{x_{q,2}^2\cdot y} \tag{8}$$
$$= \frac{(p^2+1)\cdot(\alpha+\beta p^2)}{y}$$

This results in a final form for the modeled variance estimator $\hat{V}$ in equation (8), fitting with the customary GVF forms published by the CPS: two model parameters, alpha and beta, and two inputs, *y* and *p*, that come from the standard estimate tables. In the context of earnings ratios, *y* is the universal base of the earnings percentile tables (total FTWS workers) and *p* is the ratio of the point estimates for the earnings of interest, such as female-to-male or foreign born-to-native born weekly earnings percentiles.

2.1 GVF Model Results

The following graphs visualize the replicate standard error and the GVF standard error for several series and subseries on the CPS populations of interest. When compared to the replicate standard error, the GVF series consistently smooths out noise and increases stability. Additionally, the generalized variance function standard errors retain seasonality[4] and do not remove sudden significant changes such as around 2020 with the onset of Covid-19. Figures 1 and 2 graph the entire series whereas Figures 3 and 4 graph subseries of Figure 2.

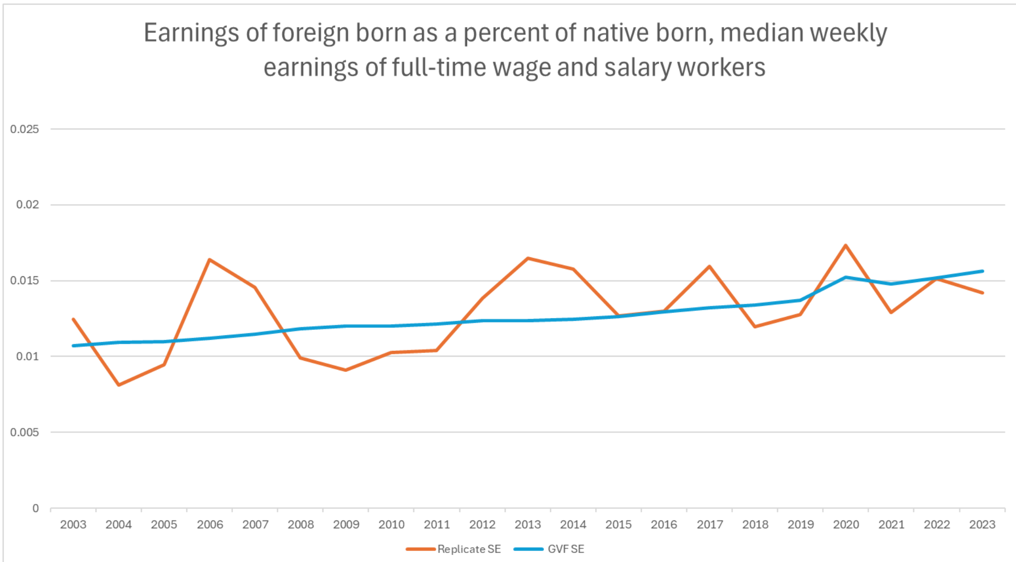

Figure 1: Standard error estimates of the ratio of median weekly earnings of full-time wage and salary workers among foreign and native born. GVF standard errors are shown in blue and replicate standard errors shown in orange.

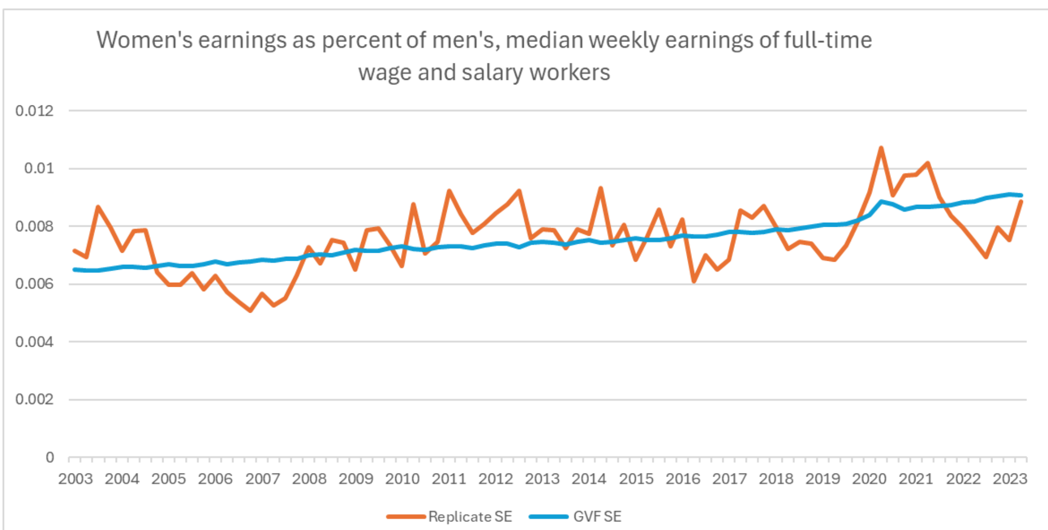

Figure 2: Standard error estimates of the ratio of median weekly earnings of full-time wage and salary workers among women and men. GVF standard errors are shown in blue and replicate standard errors shown in orange.

[4] Figures 1 – 4 in this paper present standard errors for not seasonally adjusted data. Due to the scale of the charts, seasonality in the GVF standard errors is not easy to observe but is reflected in the modeled estimates.

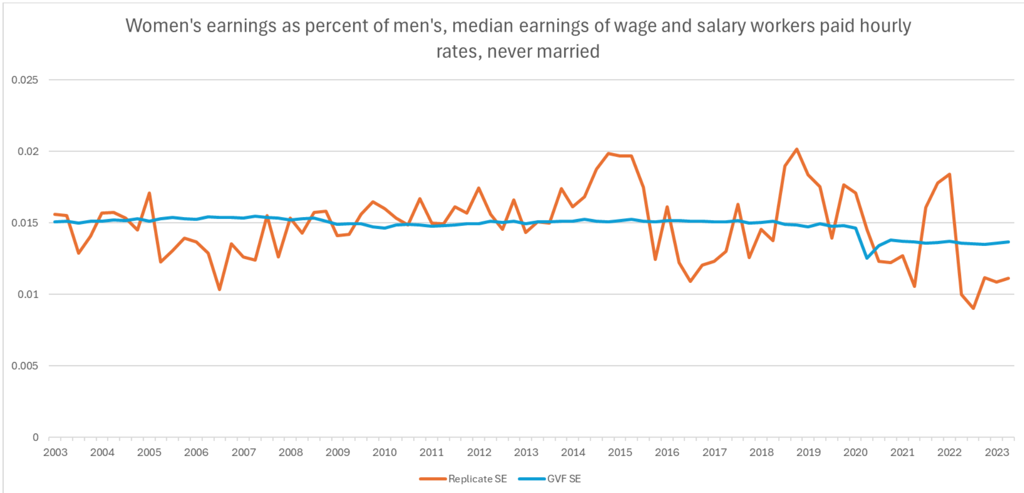

Figure 3: Standard error estimates of the ratio women's earnings as a percent of men's, median weekly earnings of wage and salary workers paid hourly rates who have never been married. GVF standard errors are shown in blue and replicate standard errors shown in orange. This figure is similar to Figure 2 but displays the subset of only those who are never married and are paid hourly rates.

While most graphs show an increase in standard error over time, the never married subgroups in Figure 3 do not. Although the response rate for the Current Population Survey has decreased, especially since 2019, the percentage of never married in the population has increased and offset any increase in standard error due to nonresponse.

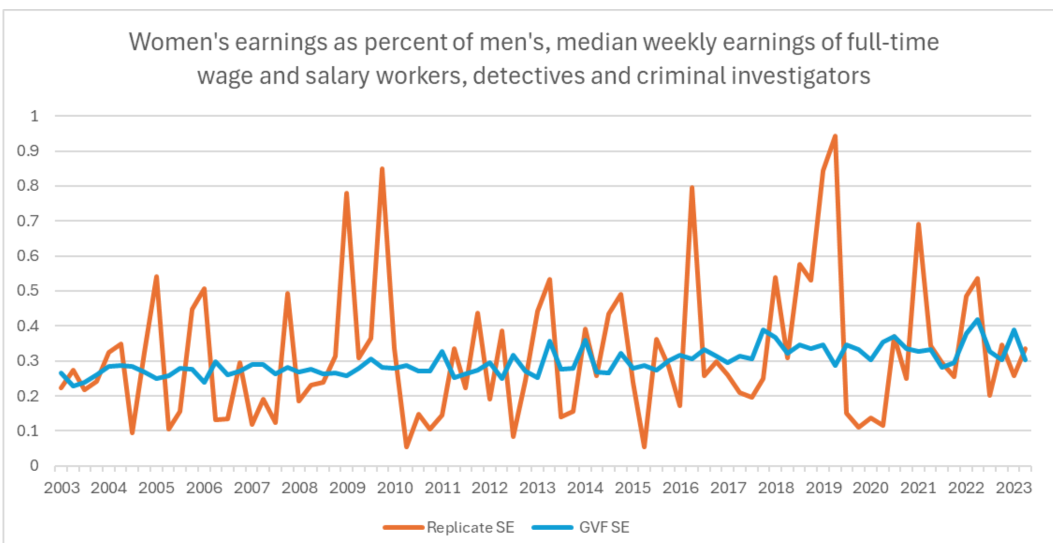

Figure 4: Standard error estimates of the ratio women's earnings as a percent of men's, hourly median weekly earnings of full-time wage and salary workers who have never been married. GVF standard errors are shown in blue and replicate standard errors shown in orange. This figure is similar to Figure 2 but displays the subset of only those who work as detectives and criminal investigators.

Subgroups such as detectives and criminal investigators in Figure 4 smooth out demonstrate how the model handles much smaller and noisier subsets and show the model is still capable of smoothing out much of the historical noise in the series.

Since BLS does not currently publish standard errors for earnings ratios, Tables 1 and 2 serve as a first look into applications of economic interest. Utilizing the publicly available point estimates combined with the GVF standard errors, these tables display the construction of 90-percent confidence intervals for the ratio of women's earnings as a percentage of men's and foreign-born earnings as a percentage of native-born, respectively. None of the presented confidence intervals contain the value one, indicating that there is a significant difference between men and women's earnings as well as between foreign-born and native-born earnings.

Median usual weekly earnings of full-time wage and salary workers by sex, seasonally adjusted						
	Women	Men	Ratio	SE	lower bound	upper bound
Q1 2022	936	1121	0.8350	0.0092	0.8198	0.8502
Q2 2022	950	1148	0.8275	0.0092	0.8123	0.8427
Q3 2022	970	1168	0.8305	0.0094	0.8150	0.8459
Q4 2022	973	1175	0.8281	0.0096	0.8123	0.8439
Q1 2023	992	1179	0.8414	0.0099	0.8251	0.8576
Q2 2023	999	1185	0.8430	0.0098	0.8270	0.8591
* BLS analyses are generally conducted at the 90-percent level of confidence.						

Table 1: 90-percent confidence intervals for median usual weekly earnings of FTWS workers by sex. This table presents seasonally adjusted estimates[5] of men and women's earnings for quarters in 2022 and 2023 and standard errors computed using GVF model (8), described in section 2.

Median usual weekly earnings of full-time wage and salary workers for foreign and native born, Not seasonally adjusted						
	Foreign	Native	Ratio	SE	lower bound	upper bound
Q1 2022	919	1067	0.8619	0.0151	0.8370	0.8867
Q2 2022	935	1072	0.8719	0.0152	0.8468	0.8969
Q3 2022	960	1096	0.8761	0.0154	0.8508	0.9015
Q4 2022	966	1116	0.8658	0.0156	0.8402	0.8914
Q1 2023	981	1124	0.8728	0.0157	0.8470	0.8985
Q2 2023	975	1125	0.8671	0.0156	0.8414	0.8928
* BLS analyses are generally conducted at the 90-percent level of confidence.						

Table 2: 90-percent confidence intervals for median usual weekly earnings of FTWS workers by nativity. This table presents not seasonally adjusted estimates of foreign-born and native-born earnings for quarters in 2022 and 2023 and standard errors computed using GVF model (8), described in section 2.

With upper bounds regularly near or below 0.90 in Tables 1 and 2, it is statistically clear that the median earnings of FTWS women and foreign born are significantly lower than for FTWS men and native born, respectively. This finding would also hold at confidence levels higher than 90 percent.

[5] The CPS program applies GVF parameters computed from not seasonally adjusted data to seasonally adjusted estimates to approximate the variance of seasonally adjusted series. This practice tends to overstate the standard errors of some seasonally adjusted series (Evans et al., 2023).

3. Ongoing Research

Of the nonstandard estimates described in the abstract, models for the earnings ratios, as described in sections 2 and 2.1, are farthest along in development, having demonstrated standard errors of suitable quality relative to the direct replicate estimates. In this sense, quality has both objective and subjective dimensions, including the computation of bias and smoothness measures as well as observational review of historical model fits for a wide variety of ratio estimates.

The bias and smoothness diagnostics are computed relative to the long-term behavior of the replicate standard error time series. On average, model (8) produces estimates with bias percentages close to zero and reasonably high smoothness percentages, depending on the size of the subgroups involved. The gains in smoothness are visually apparent in the charts in section 2.1.

Before implementation into the GVF Production System, which produces official standard errors for most CPS estimates, an effective response rate adjustment factor must be incorporated into the model. After the Covid-19 pandemic, the need for such a dynamic adjustment became apparent due to the sudden drops in response rates (BLS) that prior GVF models were not designed to accommodate. All models in the GVF Production System reflect current period response rates.

Besides the response rate adjustment for the earnings ratios, another current priority is to complete research on the GVF model for average hours at work, which is undergoing development following the general framework for mean weeks unemployed (McIllece 2018). Empirically, the quality of GVF models for the standard errors of sample means in the CPS have been negatively impacted by extreme pandemic outliers, necessitating further investigation and potentially some modifications to the underlying form of the model.

Additional expansion of GVF models for nonstandard estimates is necessary to make more variance estimates publicly available. Somes series in consideration are lost work time rate (also referred to as percent of usual work hours lost), average weekly earnings, and average hourly earnings. Developing more variance models of sufficient quality for the GVF Production System, including of many nonstandard estimates that have little research support in the general body of literature, the utility of CPS data products should be improved.

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